Batten

Binary and Multiple Systems of Stars

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This book aims to present a critical review of recent research into the subject of binary and multiple systems of stars and also to provide a survey of the study of these phenomena. In particular, it examines the theory developed on the basis of recent research into the statistical and dynamical properties of binary systems which suggests that stellar formation and the distribution of different types of stars are directly influenced by the disturbed evolution of binary components. It also contains a critical assessment of the determination of the apsidal constant. The text is written primarily for astronomers carrying out research in other fields and secondly for physicists and other scientists interested in obtaining a good background knowledge of this particular area of astronomy. It will also be useful as a supplementary text for postgraduate courses in astronomy and other related fields.

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BINARY AND MULTIPLE SYSTEMS OF STARS

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BY

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To my parents

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PREFACE

THE study of binary systems received a great stimulus a few years ago when the first computations were made of the effects of exchange of mass between the two components. Not long before this, there was a revival of interest in the statistics of binary systems, especially their distribution amongst different classes of stars. These two independent lines of investigation have converged on one result: that membership of binary systems may have had very important effects on the evolution of many stars, and the origins of several different kinds of stars may be connected with the disturbed evolution of binary components. As a consequence of this, many astronomers, and even some scientists in related fields, have become much more interested in the study of binary stars than they formerly were. When Dr. ter Haar asked me to contribute a volume on binary stars to this International Series of Monographs in Natural Philosophy, it seemed to me that such a book should most usefully be aimed at these people, who might well find it useful to have a survey of the whole field of binary stars. I have written, therefore, a series of connected review articles which certainly do not exhaust the field, but do, I believe, cover most of its boundaries with other fields in astronomy. The book is, by intention, not systematic enough to be used as a course of instruction, but I hope that in addition to its main purpose it will prove to be helpful to those who have to prepare courses on binary stars, and at least to be useful as background reading for graduate students.

Everyone has his own ideas of what should be put in to a book and what should be left out. My own choices are unlikely to win general agreement. Two principles have guided me. First, in view of the intended readership, I have assumed a fairly good knowledge of modern astrophysics, but have gone to some trouble to explain matters that are specifically within the field of binary systems. This explains the

inclusion of quite elementary material in Chapter 1—a chapter that many readers may choose to skip without great loss to their comprehension of the book. The second principle was to attempt to strike a balance between the desires to produce a self-contained book and to avoid unnecessary duplication of previously published and readily accessible work. Thus, I have not reprinted Popper's lists of reliable masses, since these are available in recent publications that most astronomical libraries possess. On the other hand, I have taken up considerable space in a discussion of reliable determinations of the apsidal-motion constant, because no other fully satisfactory discussion seemed to me to be available. Similarly, I have repeated the frequently cited, but seldom copied derivation by Tisserand of the effect of apsidal motion on the times of minima of an eclipsing system, because old volumes of *Comptes Rendus* are not readily available. The similar derivation of the effect of light time in a long-period orbit on times of minima is available in more recent publications, and is not repeated here. In general, I have not given full derivations, even of important formulae. This is partly because the emphasis of the book is on observational results and their interpretation, and partly because it seemed to me unnecessary to repeat the many derivations that can be found in Kopal's *Close Binary Systems*. Judged by these principles, I believe that neither my inclusions nor my omissions will appear wholly capricious.

I have tried to emphasize what I believe to be the essential unity of the study of binary systems, regardless of the method by which they are observed. In the last chapter, in particular, it becomes evident that the distinction between "close" and "wide" binary systems is not necessarily the same as their division into eclipsing and visual binaries. Nevertheless, much of the book is inevitably concerned with the spectroscopic observations of binaries, since that is the part of the field in which most of my experience lies.

The inclusion of multiple systems in the title may seem a little presumptuous since only one chapter of ten is devoted to them. I believe that this is the first book in which even that amount of space has been allotted to multiple systems. It may also appear that while in other

chapters my problem has been to condense how much we know into a reasonable number of pages, in Chapter 3 I succeed only in showing how little we know. The justification for including multiple systems in the title, however, is not found in Chapter 3 alone, but in the attitude which, I hope, permeates the whole book, that it is essential to take account of the known appreciable fraction of multiple systems if observations of binary systems are to be correctly interpreted. Thus there is scarcely a chapter in the book in which multiple systems are not mentioned and the effects of a third body considered.

A book of this nature is bound to be out of date, in some respects, before it appears in print. The first draft of this book was completed in the early summer of 1970, although the final version was not sent to the publishers until about a year later. Most papers published up to about the end of 1969 were perused and considered before most of the original draft was written. Papers published later than that were only included if I received some advance intimation of their contents, or they appeared to me to be important, or if for other reasons I had to rewrite the section concerned. Anything published later than mid-1971 cannot be included in the text. Inevitably, elements of chance and subjectivity have entered into these decisions, and to those who feel I have ignored important new work of theirs, I can only apologize. I am grateful to those who have made material available to me in advance of publication especially Dr. H. A. Abt, and Dr. P. Broglia whose light curve of U Cephei is used in Fig. 8.6.

Acknowledgements for permission to reproduce previously published material are due to the Astronomical Society of the Pacific (Figs. 8.3 and 9.1 and much of Chapters 8 and 9), Royal Astronomical Society of Canada (Fig. 1.4), Royal Astronomical Society (Figs. 5.1, 8.8), Società Astronomica d'Italia (Fig. 8.7), Annales d'Astrophysique (published by Centre National des Recherches Scientifiques) (Fig. 2.2), Specola Vaticana (Fig. 4.4), D. Reidel (Figs. 3.3 and 5.2) and *Sky and Telescope* (Fig. 8.1). Several other diagrams have also appeared in the *Publications of the Dominion Astrophysical Observatory*, and the spectrograms reproduced in this book were all obtained at that Observatory.

The first draft of the book was written during a very pleasant visit to the Vatican Observatory at Castel Gandolfo. I am grateful to Father D. J. K. O'Connell, S.J. (then the Director) and his staff for the hospitality they extended to me and my family in the spring and summer of 1970. I am also grateful to the Canadian Department of Energy, Mines, and Resources for leave of absence and a travel grant. The leave of absence was continued by the National Research Council of Canada, after it took over administrative responsibility for the Dominion Astrophysical Observatory in April 1970. I am especially grateful to those who have read earlier drafts of this book and offered their comments and criticisms, especially Drs. D. ter Haar, J. B. Hutchings, M. Plavec, C. D. Scarfe, A. D. Thackeray, F. B. Wood (and colleagues), and K. O. Wright. Finally it is a special pleasure to thank those whose willing co-operation has enabled me to turn a rough draft into a finished book. Dr. K. O. Wright, again, has helped and encouraged me by placing the facilities of the Dominion Astrophysical Observatory at my disposal. Miss H. D. Mann has efficiently typed the final version. Mr. S. H. Draper has prepared the illustrations. The staff and representatives of Pergamon Press, in particular Mrs. J. A. Pope, Mr. D. Webb, and Dr. ter Haar, have performed the final essential services. Drs. J. Andersen and B. Nordström have kindly helped to check the proofs, and my son, Michael, has helped me to compile the star index. To them all, I offer my thanks.

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CHAPTER 1

BASIC CONCEPTS AND TERMINOLOGY

DEFINITION AND CLASSIFICATION OF BINARY SYSTEMS

A *binary system* can be simply defined as one containing two stars that describe closed orbits around their common centre of gravity, under the influence of their mutual gravitation. No restriction has been placed in this definition on the separation of the two components. They can be in contact, or separated by thousands of astronomical units, or more. The definition would have to be stretched a little to include common-proper-motion pairs, so many of which have been discovered by Luyten (e.g. 1933), since it must for a long time remain an open question whether such pairs are moving in very long-period orbits, or simply travelling together. This book is almost exclusively concerned with pairs that show a discernible orbital motion in about a hundred years, at most. This effectively limits consideration to pairs with separations of up to ten or a hundred astronomical units. For the most part, the book is concerned with pairs of very much smaller separation.

A class of objects in which one basic parameter is allowed to vary over such a wide range will contain many dissimilar individuals, and it is natural to look for some form of classification that will divide the whole group into more nearly homogeneous sub-classes. An obvious classification is provided by the methods of observation. Very wide pairs can be resolved by the telescope and be recognized visually as double stars, if they are near enough to us. These are the *visual binary systems*. It is convenient to include in this class pairs that are not actually resolved, but which are discovered by careful measurements of the position of a star relative to background stars in the same field. These systems are usually termed *astrometric binaries*. Many binary

systems, however, contain stars that are so close together that they can never be resolved in a telescope, or be discovered from their transverse motions. They can be discovered spectroscopically from the variations in their radial velocities, and are therefore called *spectroscopic binaries*. The orbital planes of some of these are oriented so that the two components eclipse each other as seen from the Earth, and these are called *eclipsing binaries*. There are two other groups of objects whose binary nature is less directly inferred. The *spectrum binaries* are stars that show composite spectra, but whose components are too widely separated from each other to show observable velocity changes; some of these might be visual binaries if they were nearer to the Sun. Some, at least, of the *ellipsoidal variables* are binaries that just fail to be eclipsing, but show periodic light variations because of the distortion of their components.

The techniques used to observe these different kinds of binary are very different, and a classification based on them is very useful for distinguishing the astronomers who study binary systems. It has few other merits, however. Recognition of a visual binary depends not only on the true separation of its components, but also on its distance from the Sun. Visual binaries, at least those for which orbits may be determined, are relatively near neighbours. Spectroscopic binaries can be recognized at much greater distances, since it is only necessary for their discovery, that the system be bright enough for spectrograms of a reasonably high dispersion to be obtained. Eclipsing variables can be recognized at even greater distances if the eclipses are fairly deep. Such binaries have been recognized in another galaxy (Baade and Swope, 1965). Another objection to a classification based on method of observation is that it is not unique. All eclipsing binaries are at least potentially spectroscopic binaries. About a hundred visual binaries can also be profitably observed spectroscopically. A few (ε Aurigae and VV Cephei for which astrometric orbits have been published) can be observed in all three ways.

Schemes of classification have been proposed by Krat (1944), Struve (1950), and Sahade (1960). All these depend on the position of the component stars in the Hertzsprung–Russell diagram, and

Sahade's scheme will be discussed as the type of this classification. He recognizes five groups of binary systems: (i) those in which at least one component is contracting towards the main sequence (Fig. 1.1); (ii) those in which both components are on the main sequence, group (ii) is subdivided into (iia) similar components and (iib) dissimilar components; (iii) systems containing one component on the main sequence, and one that is a giant or subgiant; (iv) systems in which

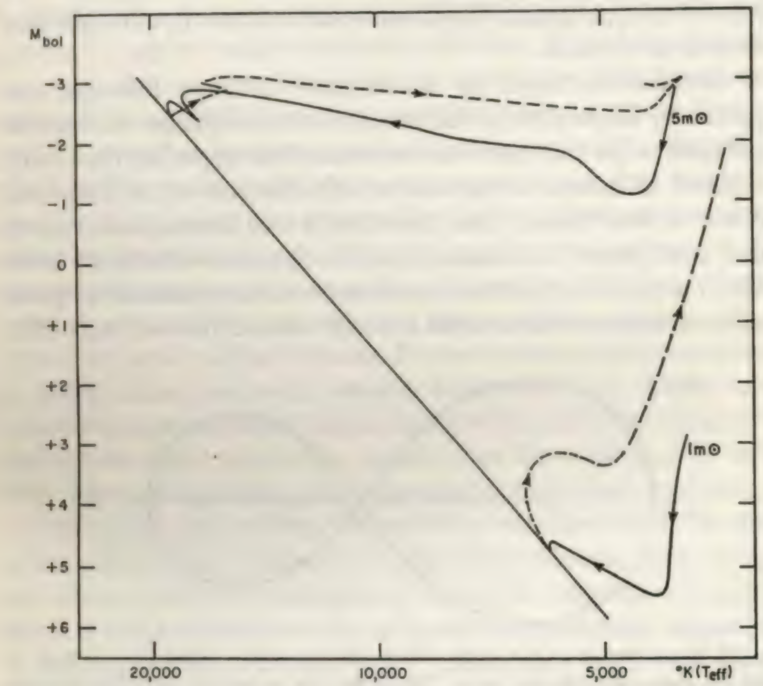

FIG. 1.1. Approximate evolutionary tracks in the Hertzsprung–Russell diagram of stars of $1m_\odot$ and $5m_\odot$. The straight line represents the theoretical main sequence. Continuous lines show evolution towards the main sequence (contraction) and the dashed lines evolution away from the main sequence (expansion). The ordinates are absolute bolometric magnitude, and the abscissae are effective temperature. The more massive star has a spectral type of about B3 when on the main sequence, and about K1 at the extreme right-hand ends of its track. Its radius at this spectral type must be about 30 times the radius it has on the main sequence, since its luminosity is greater at the cooler spectral type.

both components are giants or subgiants, also divided into (iva) and (ivb) in the same way as group (ii); and (v) systems with at least one component below the main sequence. An important group, the W Ursae Majoris systems, does not fit into any of these classes, and really forms a sixth class by itself. This system of classification is of great value in pointing out evolutionary relationships between the components in a system, but it does suffer from disadvantages. For instance, the systems containing Wolf–Rayet stars are found together with the dwarf M-type system YY Geminorum in class (i), although they have little in common.

A classification based on a completely different principle was proposed by Kopal (1955). The presence of a companion star sets an upper limit to the size that a star can reach. This upper limit is usually considered to be one of the zero-velocity surfaces of the restricted problem of three bodies. The derivation is well known, and is given with a description of the surface itself in chapter 3 of Kopal's book (1959). The particular surface is the lemniscoid that contains one point of contact between the volumes enclosed around the two stars (Fig.

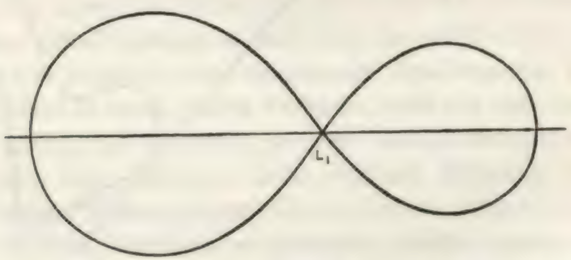

FIG. 1.2. Cross-section of the Roche lobes with the orbital plane for a system in which the mass ratio (m_2/m_1) is 0.5. The larger lobe surrounds the more massive star. The point L_1 is the *inner Lagrangian point* (or often, the *Lagrangian point*).

1.2). Particles that cross that surface no longer "belong" to the star within the lobe they have just left, but are subject to the gravitational forces of both components, and may eventually leave the binary system. The relative dimensions of this surface are completely specified by the ratio of masses of the two stars. The surface is frequently

referred to as the "Roche limit", although some authors, probably to avoid confusion with the Roche limit of satellite theory, prefer other phrases such as "Lagrangian surface", or "limit of stability". In this book, the surface is called the *critical surface*, and the two volumes enclosed by it are called the *Roche lobes*. Kopal's classification divided all binary systems into three groups: (i) systems in which neither component fills the Roche lobe [*detached* systems, roughly equivalent to Sahade's class (ii)]; systems in which only one component fills its Roche lobe [*semi-detached* systems, included in Sahade's class (iii)], systems in which both components fill their Roch lobes (*contact systems*, including, according to Kopal, the W Ursae Majoris systems, although Wood (1969) has questioned this). The use of this form of the critical surface has been criticized on two counts: the derivation of the surface assumes that the two stars are mass points moving in circular orbits, and it also assumes that only gravitational forces are acting on the particles in the outer layers of the stars, whereas stellar atmospheres are known to be subject to complex hydrodynamic and magnetic effects. The justification for the use of this representation of the critical surface is that it works. No well-observed system is known in which either component exceeds substantially its Roche lobe; several are known in which there is strong evidence that one component fills its lobe, and it is just these systems that show evidence of instability. Many of them (the *Algol-type* systems) show deep total eclipses and the relative radii of the stars are fairly well determined. The mass ratios, unfortunately, are not well determined, because the secondary spectrum is visible only during the eclipse of the bright star. Nevertheless, it is well established, statistically, that the secondary components of Algol systems (and of some other systems too) do fill their Roche lobes, and this suggests that the assumptions made in deriving the critical surface, although admittedly made with more regard to mathematical convenience than to physical reality, are not far from the truth. In Chapter 6, empirical evidence is presented that the concentration of matter towards the centre of a star is indeed high, particularly for subgiants, so the first assumption has a measure of observational backing. Reuning (1970) has considered the consequences of dropping

this assumption, and finds that only a slight modification of the shape of the star results. It is not known what the consequences of dropping the second assumption would be, but the observations indicate that they would not be serious.

Kopal's scheme has the advantage that it employs information such as the masses and radii of the component stars to classify binary systems. It ignores the information on relative luminosities that Sahade's scheme was designed to use. Plavec (1964) has suggested that the two schemes could well be combined into a two-parametric classification of binary systems. A complete specification of a binary system needs seven parameters: the mass, radius, and luminosity of each component, and the mean distance between maximum and minimum separation. It is thus not surprising that no generally accepted or comprehensive system of classification has yet emerged. The combination of Sahade's scheme and Kopal's, as suggested by Plavec, does make use, implicitly, of all seven parameters, and probably represents the best classification suggested to date.

One of the parameters, the separation between the two components, is of overriding importance. A distinction is very frequently drawn between "wide" and "close" binary systems. Until recently, "close" binary systems were considered to be those in which the two components are close enough to each other to distort each other. In this sense, virtually all visual binaries, and many spectroscopic binaries, are "wide". Only binaries with periods of a few days or less can be considered "close". At a conference held in 1966, however, both Plavec (1967) and Paczynski (1967) independently proposed a new definition of the term "close binary system". They defined "close" system as one in which one component affects the evolution of the other. This definition is intimately bound up with Kopal's scheme of classification, because the effect of normal stellar evolution is to increase the radius of the evolving star very considerably. At a certain stage of its evolution, therefore, the natural radius for a component of a binary system may very well be greater than that of its Roche lobe, even if, as a main-sequence object the star was much smaller than the lobe. In such a system, the two stars will affect each other's evolution.

The detailed discussion of this matter belongs to Chapter 10, but it should be pointed out here that very many binary systems must be "close" in this sense. Indeed, Plavec (1968) has suggested that nearly all binary systems must have contained components that exceeded their Roche lobes in the pre-main-sequence stage of evolution. Even if this stage is not considered, it is now known that, after the exhaustion of hydrogen, helium, and carbon in its core, a star will expand so much that even some visual binaries should be regarded as "close". Probably some stars expand even more when they have exhausted still other nuclear fuels. On the other hand, stars of very low mass do not expand when they leave the main sequence, but evolve directly into degenerate dwarfs without ever burning hydrogen (Kumar, 1963). Two stars of this kind could be quite close to each other without the evolution of either being affected, and a binary system containing them would be considered "wide", even if it had a period of about a day. Such a binary would be hard to discover, and none are known. A few binaries containing two M-type dwarfs are known (YY Geminorum and possibly CC Eridani) but these are probably massive enough to expand eventually, although not for a very long time. This definition of "close binary" directs attention to the whole evolutionary history of a binary system, and shows that very many systems must be regarded as "close". The variety of binary systems that can be observed is probably largely a result of the fact that in different systems one of the components (the more massive, as is shown in Chapter 10) fills its Roche lobe at different stages of its own evolution, and the subsequent development of the binary system depends on this.

ORBITAL ELEMENTS

The term *orbital elements* originated in the study of planetary motions. Six quantities are needed to define the motion of a planet in its orbit, and those usually chosen are (Fig. 1.3):

(i) Two angles i, Ω which define the position of the orbital plane in space with respect to a reference plane. The choice of plane

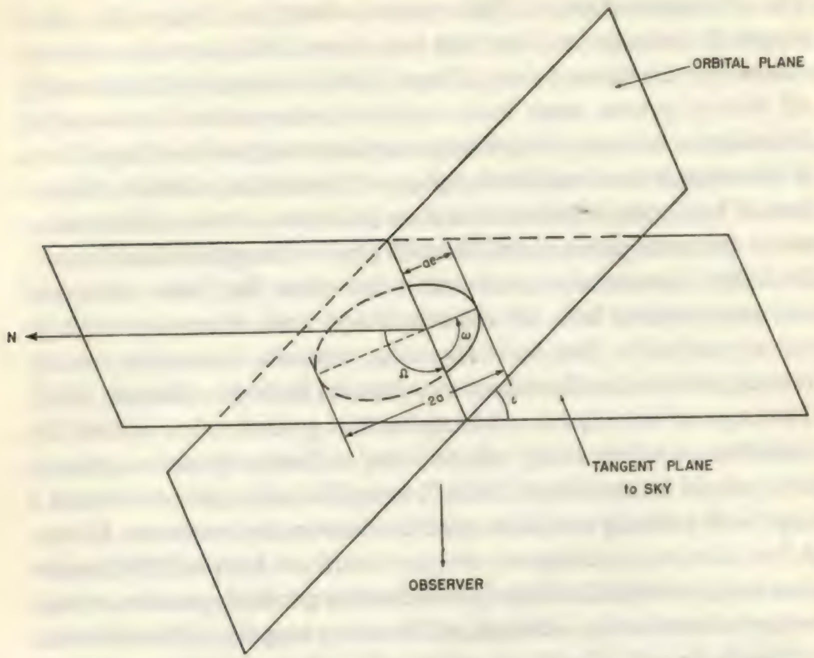

FIG. 1.3. Orbital plane and the tangent plane to the sky illustrating the significance of the orbital elements.

is arbitrary; for planetary orbits it can be that of the Earth's orbit, or the invariable plane of the Solar System.

(ii) a third angle ω, defining the orientation of the longest axis of the elliptical orbit within the orbital plane. In many dynamical studies of the Solar System, it has been found convenient to use another angle $\tilde{\omega}$, defined as $\omega+\Omega$. This composite angle is called the *longitude of perihelion:* note that it is composed of two angles in *different* planes.

(iii) two quantities a (the major semi-axis) and e (the eccentricity) that define the size and shape of the ellipse.

(iv) a time T corresponding to a given position of the planet in its orbit (usually the time of perihelion passage). The period, P,

is not a necessary separate element, since within the Solar System all periods are given by Kepler's third law

$$\frac{a^3}{P^2} = \text{constant.}$$

The orbit of a binary system may be similarly defined although a different reference plane is chosen, and the period is related to the major semi-axis by the generalized form of Kepler's law

$$\frac{a^3}{P^2} \propto m_1+m_2$$

where m_1 and m_2 are the masses of the two stars. A complete dynamical description of the system, therefore, requires a knowledge of the period P, which has come to be regarded as an extra orbital element. The following quantities are used to define a binary orbit:

P = the orbital period, usually expressed in days (spectroscopic or eclipsing binaries) or years (visual binaries).

i = the inclination of the orbital plane to be the tangent plane of the celestial sphere at the star.

Ω = the position angle (measured from north through east) of the line of nodes joining the intersections of the orbital and tangent planes, and measured in the latter.

ω = the angle between the direction to the ascending node (at which the star crosses the tangent plane while receding from the observer) and that to the point of closest approach of the two stars (periastron). This angle is measured in the orbital plane, in the direction of orbital motion. By convention visual observers always give the value of ω appropriate to the secondary (fainter) component of a system, while observers of eclipsing and spectroscopic systems usually quote the value for the orbit of the primary component. In any given system, these two differ by 180°. This angle is usually called the *longitude of periastron.* The term is a little unfortunate, because it

suggests an analogy with the longitude of perihelion. That angle, however, is measured in two different planes, whereas the longitude of periastron is measured in one plane only. The point is made here, because the two angles ω and $\tilde{\omega}$ have been confused in some published dynamical investigations of the orbits of binary systems.

a = the major semi-axis of the orbit, usually expressed in kilometres or astronomical units.

e = the eccentricity of the orbit, a dimensionless number between zero and unity.

T = the time at which the two stars pass through periastron.

From observations of a visual binary the quantities P, i, ω, e and T can be determined. It is not possible, however, to distinguish between the ascending and descending nodes of the orbit unless radial-velocity observations of the stars are also available. There is, therefore, an ambiguity of 180° in the value of Ω derived from visual observations alone. It is conventional to take $0° \leq i \leq 90°$ if the apparent motion is direct, and $90° \leq i \leq 180°$ otherwise, and to assume that the node for which $\Omega < 180°$ is the ascending node, unless there is evidence to the contrary. The apparent major semi-axis a'' (seconds of arc) can be derived from the visual observations but it cannot be converted into an absolute value of a unless the distance can be independently determined. If radial-velocity observations have been made, they can be used to give the scale of the orbit, and thus to determine the distance. The major axis, and hence the distance, can be estimated by the use of Kepler's third law if the masses of the component stars are assumed to be normal for their spectral types. Such an estimate of distance, when expressed as a parallax, is usually known as a *dynamical parallax*.

Observations of a spectroscopic binary yield rather different information. The period is readily determined, but neither Ω nor i can be determined from spectroscopic observations alone. Interferometric observations of spectroscopic binaries, or observations of the occultations of spectroscopic binaries by the Moon can give this information, but these methods are in their infancy, and the concern here is with information that can be derived from spectroscopic observations. The elements e and ω can be determined from the shape of the velocity curve (that is the plot of the variation of velocity with time reduced to a single period, Fig. 1.5). Two (or in some circumstances three) other quantities can be determined:

V_0 = the radial velocity of the centre of mass of the system. (This is also often denoted by γ.)

K_1 = half the total range of the radial-velocity variation of the brighter star.

K_2 = half the total range of the radial-velocity variation of the fainter star. (This can only be determined if the fainter star is bright enough for its spectrum to show in the combined light of the system.)

These three quantities are all velocities, and are normally measured in kilometres per second. The period of a spectroscopic binary is usually measured in days. Given these units, the values of K_1 and K_2 are related to the orbital elements by

$$a_{1,2} = 13{,}751\,(1-e^2)^{1/2}K_{1,2}P \quad \text{km},$$
$$m_{1,2} = 1.0385\times10^{-7}(1-e^2)^{3/2}(K_1+K_2)^2K_{2,1}Pm\odot$$

where

$$a_1+a_2 = a,$$
$$a_2/a_1 = K_2/K_1 = m_1/m_2.$$

Thus, K_1+K_2 is a measure of the major semi-axis projected onto a plane that contains the line of sight. If only one spectrum is observed, then only $a_1 \sin i$ can be determined, and the only information about the mass is the so-called mass function

$$f(m) = \frac{m_2^3 \sin^3 i}{(m_1+m_2)^2} = 1.0385\times10^{-7}(1-e^2)^{3/2}K_1^3Pm\odot.$$

If the fainter star (component 2) is also the less massive, then it can easily be shown that the minimum values of $m_{1,2} \sin^3 i$ are given by

$4f(m)$. By extension of the ideas developed for the description of planetary orbits, the quantities K_1, K_2, and V_0 are usually regarded as elements of the orbit of a spectroscopic binary.

The situation is different again for an eclipsing binary. The period can of course be determined. The angle i can be found, because it is related to the minimum distance between the centres of the two stars, projected onto the plane of the sky. This quantity affects both the shape and depth of the light curve (a plot of light intensity against time, reduced to a single period). The direction of travel of the occulting stellar disk as it impinges upon that of the eclipsed star cannot be found from the light curve, and the angle Ω is indeterminate. If both eclipses are observed, $e \cos \omega$ can be determined from the displacement between them, expressed as a fraction of the period. If $e \cos \omega = 0$ (circular orbits, or elliptical orbits of which the major axis lies along the line of sight) secondary eclipse (that of the fainter star) will occur exactly half-way between two successive primary eclipses. For all other values of $e \cos \omega$ the time of mid-secondary eclipse will be displaced by an amount proportional to $e \cos \omega$ (to a first approximation when e is small). If both eclipses are very well observed, they will, in general, be found to be of different durations, and the difference, again to a first approximation when e is small, is proportional to $e \sin \omega$. Thus, in principle, e and ω can be determined for the orbit of an eclipsing binary. In practice, however, $e \sin \omega$ cannot be determined very accurately from the difference in duration of the two eclipses, although a better result can be obtained from a study of the shape of the whole of the light curve during both eclipses.

Nothing can be learnt from the light curve about the absolute size of the orbit, and the element a remains unknown unless spectroscopic observations are available. The shape and duration of the eclipses, however, depend on the radii of the two stars, expressed as fractions of their separation, and these two fractional radii can therefore be determined from the light curve. They are frequently regarded as "elements" of an eclipsing binary. If the system is also observed spectroscopically, then it is possible to determine the radii of the stars in absolute units.

This completes the geometrical knowledge that can be obtained about the different kinds of binary system. Further physical information can be obtained, however. The spectral types are important information, and it would be good if observers adopted the habit of giving spectral type, some estimate of absolute magnitude, apparent magnitude, and U, B, V colours with the orbital elements. Another important item is the relative brightness of, or magnitude difference between, the two stars. For a visual binary whose components are sufficiently separated, this can be measured by normal photometric methods. Unfortunately, the components of most of the interesting visual binaries (those with orbital periods short enough for the elements to have been obtained) have such small angular separations that accurate measurement of their magnitude differences was impossible until the recent development and application of the Rakos area scanner (Franz, 1967). For spectroscopic binaries, the relative intensities of the two component spectra provide a clue. Ordinarily, if the two stars differ by more than a magnitude or a magnitude and a half in brightness, the spectrum of only the brighter one will be seen. If both spectra are visible, the relative intensities of their lines are, under certain conditions, directly proportional to the luminosities of the two stars. Although this was long recognized, methods for the quantitative measurement of the magnitude difference were developed by Petrie (1939, 1950). His methods are based on spectrophotometric measurements of the intensities of selected spectral lines. They may break down, however, if the two stars are of widely different spectral types, or if for some other reason the line profiles in the spectra of the two stars are of different shapes. There can also be difficulties if the lines are so broad that they cannot be properly resolved. All these sources of error are discussed by Petrie in his original papers. The results of such spectrophotometric methods are usually expressed as a difference of *visual* magnitude, Δm, between the two components. What is actually measured, however, is the relative depths, or equivalent widths, of absorption lines in the spectra of the two components.

It is also possible to determine relative luminosities of the components of an eclipsing binary. If one of the eclipses is total, the determina-

tion is simple because the depth of totality is easily obtained from the difference in magnitude between the two stars. If the eclipses are partial, the quantity directly obtained from the observations is the ratio of surface brightnesses of the two stars, because the same area of each star is eclipsed (unless the orbit is elliptical) and the ratio of the depths of the two eclipses is that of the surface brightnesses. The luminosities follow when the radii of the two stars are determined from the shape of the light curve. The result is normally expressed as a ratio of luminosities at some specified wavelength, rather than as a magnitude difference. When surface brightnesses can be measured in several wavelengths, the photometric observations can be used to determine the difference in spectral types, thus valuably supplementing or confirming the spectroscopic data.

Another quantity that can be determined from a light curve is the degree to which the light of the eclipsed star is darkened towards the limb. In principle this might seem to be easy, because it is obvious that the limb darkening will affect the shape of the light curve. Unfortunately, it has its maximum effect only over a relatively small portion of the light curve, when the limb regions of the eclipsed star are being covered or uncovered, and the accurate determination of limb darkening has proved to be a very difficult matter. This problem is discussed more fully in Chapter 7.

It is also possible in principle to determine the amount of deformation of the components of a close binary system from the variation of light between eclipses. An ellipsoidal star displays more surface at quadratures than at conjunction. Light variations between eclipses are indeed observed, but they are complicated by light from one star that is intercepted and reradiated by the other ("reflection") and by the presence of matter between the stars (Chapter 8). Moreover, the simple variation in light to be expected from the variation in surface area presented to the observer is modified by the presence of gravity darkening (at least in some stars), and any quantitative information that may be obtained about the distortion of the stars in a binary system is necessarily of low weight and suspect.

The contents of this section are summarized in Table 1.

TABLE 1. SUMMARY OF INFORMATION OBTAINABLE FROM BINARY SYSTEMS

Element		Visual binary	Spectroscopic binary		Eclipsing binary
			One spectrum	Two spectra	
	P	✓	✓	✓	✓
	a	Apparent a''	$a_1 \sin i$	$a \sin i$	no
	e	✓	✓	✓	✓
	ω	✓	✓	✓	✓
	T	✓	✓	✓	✓
	i	✓	no	no	✓
	Ω	✓*	no	no	no
	m_1	if parallax	$f(m)$	$m_1 \sin^3 i$	no
	m_2	known		$m_2 \sin^3 i$	no
Radii	R_1	no	can be estimated from knowledge of spectrum and luminosity		$r_1 = (R_1/a)$
					r_2
Fractional luminosity	L_1	✓	can be estimated from knowledge of spectrum		✓
	L_2	✓			✓
Spectral types		✓	✓	✓	if several colours available
Limb darkening	u_1	no	no	no	in principle
	u_2	no	no	no	in principle
Ellipticity		no	no	no	in principle

* Ambiguous without radial-velocity observations.

DETERMINATION OF ORBITAL ELEMENTS

Detailed instructions are not given in this book for the computation of orbital elements of binary systems that have been observed in any of the three different ways. A manual of all methods of orbit determination would certainly be very useful, especially if it were a self-contained monograph. Probably no one person could write it satisfactorily, since few, if any, have had experience of computing orbital elements for spectroscopic, visual, and eclipsing binaries. Indeed, there are not many people with extensive experience in any *two* of these fields. In this section, only some general principles that have

application to all three problems are discussed, and references are given to more detailed discussions of the various methods. The period of the system is assumed to be known. A fuller discussion of the orbital period and its determination is given in Chapter 4. It is usually convenient, in practice, to determine the orbital period separately from the other elements, although it is possible, in theory, to determine it with the other elements. Unless care is taken, however, a consistent set of elements may be obtained, of which the only disadvantage is that it is completely wrong.

Although the observed quantities are very different for the three kinds of binaries, the various methods of solution for the orbital elements have developed in similar patterns: first graphical methods have been devised, then analytical methods, usually based on the principle of least squares. Graphical methods have the advantage of speed, but least-squares methods usually do more justice to the observations and also provide a measure of the uncertainties of the elements. Without some measure of uncertainty, no set of elements is complete. On the other hand, one should beware of the attitude that regards "least-squares" as a magic box into which one can pack the observations and squeeze out the answers. The availability of fast computers that can be programmed to handle large numbers of observations without forming normal points perhaps makes this attitude easier to adopt. It is hardly necessary to think about an orbital solution any more. The principle of least squares can be applied only when the observational errors are distributed according to the normal law, and when the equations of condition are linear in the unknowns. Systematic errors in any considerable number of observations will undoubtedly vitiate the determination of the uncertainties, and possibly that of the elements themselves. The restriction to linear equations of condition implies that the least-squares solution must be for differential corrections to values of the elements already approximately known. It is not an appropriate method for the determination of the first approximate elements of a new system. Much work now, at least on spectroscopic and eclipsing binaries, is concerned with obtaining second-epoch elements, and the first-epoch elements can nearly always be

used as the approximate elements to be improved by a least-squares method. When a new system is being investigated, however, some method of preliminary orbital determination is needed, and graphical methods still have a role to play.

These principles have been well understood and practised in the study of visual and spectroscopic binaries. Even when all precautions are observed, however, there are circumstances in which least-squares solutions can break down. A well-known example is the case of a spectroscopic binary with an orbit of small eccentricity. The Lehman-Filhés normal equations (1894) then fail because the coefficients of the corrections $\Delta\omega$ and ΔT are almost proportional to each other and the equations become indeterminate. Sterne (1941) derived alternative methods of solution, one of which is particularly designed for systems with nearly circular orbits. In this method one determines $e \sin \omega$ and $e \cos \omega$ and the epoch T_0 of maximum velocity of recession, instead of the quantities e, ω, and T. The linear dependence of two sets of coefficients in the normal equations is then avoided. A less obvious example is the interaction between the element V_0 and the other elements. All methods of determination of the elements of a spectroscopic binary depend on expressing in differential form the equation

$$V = V_0 + Ke \cos \omega + K \cos (v+\omega),$$

where V is the observed radial velocity, and v is the true anomaly in the orbit. The right-hand side of this equation contains two constant terms, V_0 and $Ke \cos \omega$, and it is obvious that an incorrect determination of V_0 will introduce errors in the determination of K, e, and ω. This can happen if the distribution of observations is unfavourable, even if there are no important systematic errors in the observations themselves. Similarly, if e and ω are badly determined and the term $Ke \cos \omega$ is large, an appreciable error may be introduced into the determination of V_0. Petrie has suggested (1962a) that this effect may account for some of the cases in which different values of V_0 are found for the two components of a spectroscopic binary system. He found that it is indeed a possible explanation for the system H.D. 209481.

The situation is much more complicated for eclipsing binaries. The basic equation that connects the geometrical elements r_1, r_2, and i with the observed loss of light α is an implicit equation in the phase angle θ. The equation is

$$\sin^2 \theta \sin^2 i + \cos^2 i = r_2^2\{1 + kp(k, \alpha)\}^2,$$

where $k = r_2/r_1$, the function $p(k, \alpha)$, called the geometrical depth of the eclipse, is given by

$$p = (\delta - r_2)/r_1,$$

and $\delta = \delta(\theta)$ is the projected distance between the centres of the two stars. Thus any method of solution, whether graphical or least-squares, must be iterative. For this reason, there has been a sharp difference of opinion amongst astronomers as to the applicability of least-squares methods to the solution of eclipsing-binary orbits. On the one hand, the original graphical methods of Russell and Shapley (1912) have been brought to a high pitch of refinement with the publication of Merrill's nomograms (1950, 1953). On the other hand, Kopal and Piotrowski have developed numerical methods that need no more graphical interpolation than is necessary to determine the depths of the two minima and the times of the contacts. Shortly before his death, Wyse (1939) was developing methods of finding differential corrections to graphically determined elements. This seemed a promising method at the time, since it was based on a close analogy with the methods used for spectroscopic binaries. Kopal has argued very vigorously for the elimination of the graphical step, or at least the interposition of his numerical method between the graphical step and the differential corrections. Many observers, however, have been reluctant to apply least-squares methods to any but the best observations, feeling that the labour is hardly justified except in such cases. Kopal has also developed the method of differential corrections and published important tables for it (1947).

Another difficulty is the need for "rectification" of a light curve. It has already been explained that the light of a binary system can vary

between eclipses because of the deformation of the component stars, and the re-radiation (or "reflection") of the light of one component by the other. The first cause makes the system appear brighter at quadratures than it does just before or after eclipses. The second, if the two stars differ considerably in luminosity, makes the system appear brighter near the time of secondary eclipse than it does near the time of primary. These changes have to be removed from the light variations observed during eclipse if the orbital elements are to be successfully determined, and the process of removal is termed *rectification*. Unfortunately, many systems show changes in their light that cannot be ascribed to either of these causes. They must rather be due to intrinsic variations in one of the component stars, or else to the interference of gas streams with the light of the system. As the accuracy of photometric observations increases, rectification becomes more and more an uncertain procedure.

These considerations led Kopal (1962) to suggest an entirely new method based on the Fourier transform of the light curve. In this way, he hoped to separate the proximity effects that depend on harmonics of the orbital period from the eclipses, and from the "high-frequency" observational errors. This suggestion stimulated work by Kitamura (1965) on the use of incomplete Fourier transforms of the light curve. His method, which is probably well adapted to use on a high-speed computer, does not entirely eliminate rectification but does provide a check on the process that was missing from others. Kitamura has also published the extensive tables needed (1967). A similar method has been developed by Mauder (1966) who used complete Fourier transforms, and was able to eliminate separate rectification.

All these methods may be superseded by new ones now being developed. Several investigators are studying the determination of elements by the comparison of the observations with numerical models of binary systems. Their work has only recently been made possible by the availability of more powerful computers. The effects of distortion and reflection are built into the model, in accordance with the theory of stellar atmospheres. The methods are discussed in more detail in Chapter 7.

The numerical and practical details of the determination of orbital elements can be found elsewhere. Good descriptions of the determination of the orbit of a visual binary have been given by van de Kamp (1958) and van den Bos (1962a). The computation of the orbit of a spectroscopic binary has recently been described by Petrie (1962b). Sterne's own description of his methods has already been cited. Kopal's methods for eclipsing binaries are fully described in his book (1959). The Russell–Merrill method, and Kitamura's and Mauder's methods are fully described in the references already given.

Catalogues of orbital elements of binaries have been published. A catalogue of the elements of visual binaries has been prepared by Finsen and Worley (1970): it contains elements for 696 systems. It replaces an earlier catalogue by Worley (1963) which is more frequently referred to in this book, because it was the only one available at the time of writing. The most recent catalogue of elements of spectroscopic binaries has been compiled by the present writer (1968a). It contains elements for 737 systems. A catalogue of elements of 211 eclipsing binaries has been published by Koch *et al.* (1970). It is hoped shortly to supplement these catalogues by a catalogue of reliable absolute dimensions.

SIGNIFICANCE OF ORBITAL ELEMENTS

What confidence can be placed in a newly determined set of orbital elements? It is shown in the previous section that a systematic error in one element is inevitably reflected in errors in the other elements. An illusory accuracy can sometimes be obtained if the determination of too many unknowns is attempted at once. A nearly circular orbit should probably be treated as circular although the introduction of two extra parameters e and ω will nearly always improve the fit to the observations. A good criterion for the reliability of orbital elements is their repeatability. Truly independent determinations of orbital elements at different epochs have been made for surprisingly few systems. For example, among spectroscopic binaries, only about one quarter has been observed at more than one epoch.

About half that number show some evidence of significant or suspected changes in the "orbital elements" (Batten 1968b), most of which are of such a nature that they cannot be real changes in the binary system. Periodic variations in V_0 can be attributed to the presence of a third body, and a regular advance in the value of ω is usually considered to be an effect of the internal structure of the component stars, although it can also be influenced by the presence of a third body and, of course, by relativistic effects. Large irregular fluctuations in K and e cannot be so easily explained. Because e is often poorly determined, it is natural to focus attention on the element K, in which these apparent changes are usually found. For some systems (e.g. AR Cassiopeiae and δ Capricorni) they would imply *fluctuations* in the total mass of the system, if they were interpreted at their face value. Many of the first-epoch observations were obtained with low spectrographic dispersion, and this will undoubtedly have increased the photographic and measuring errors that can produce systematic effects in K discussed in more detail in Chapter 5. This is especially true for those spectroscopic binaries in which the spectra of both components are visible—but the two examples given above are both binaries that exhibit the spectrum of only one component. A failure of the method of solution such as that discussed in the previous section may result in erroneous elements being derived for the two epochs. This is quite likely if the observations are not uniformly distributed over the velocity curve. Orbital elements may also be affected by the pulsation of one of the components, or by the presence of an undetected third body. The element ω (and e) as derived from spectroscopic observations can be very different from the value derived from the light curve of the same system. There is little doubt that the source of the error is in the velocity curve. Struve interpreted this as evidence that in many systems the stellar spectra are contaminated by the spectrum of a gaseous stream. From this point of view, the problem is discussed in more detail in Chapter 8. In Fig. 1.4 the latest representation of the distribution of ω, derived by Batten and Ovenden (1968), is shown. It is very far from the uniform distribution to be expected if orbits are oriented at random in space, and therefore many values of ω are suspect.

From all this information, it seems that the cautious view is to regard orbital elements of spectroscopic binaries as parameters that define more-or-less well the observed velocity variation at a given epoch, rather than as measures of the physical properties of a given

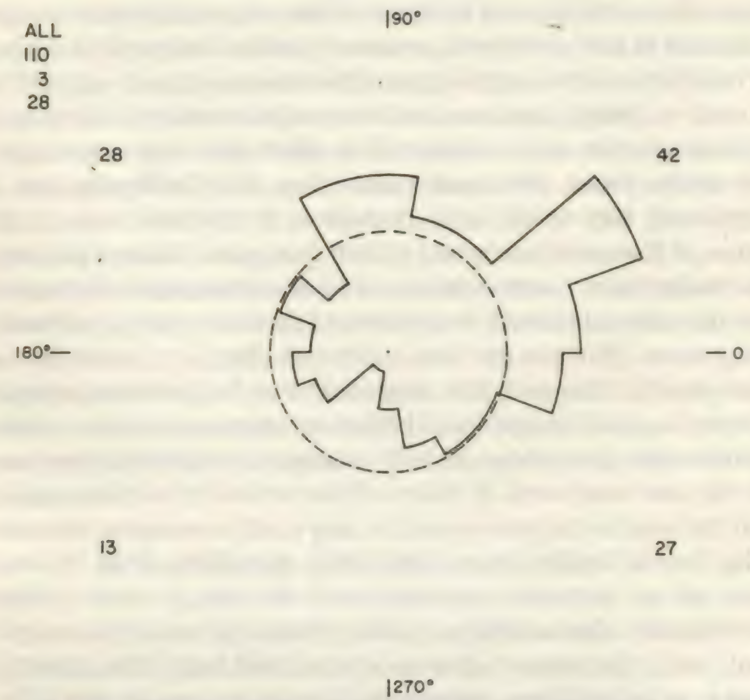

FIG. 1.4. Distribution of ω for all systems in the *Sixth Catalogue of the Orbital Elements of Spectroscopic Binary Systems* that have a determined orbital eccentricity. Numbers in each corner indicate the total numbers of systems in each quadrant. Numbers at top left are: total, average per 10° interval (broken circle), and average for 90° interval.

binary system. Only when two or three independent sets of observations give substantially the same values for the orbital elements can any certainty be felt that the true elements of the system are known. This may be a counsel of perfection, but it is certainly desirable to

attempt to determine the elements of many more systems at more than one epoch.

There is also considerable evidence for changes in light curves from one epoch to another. This is discussed in more detail in Chapter 8.

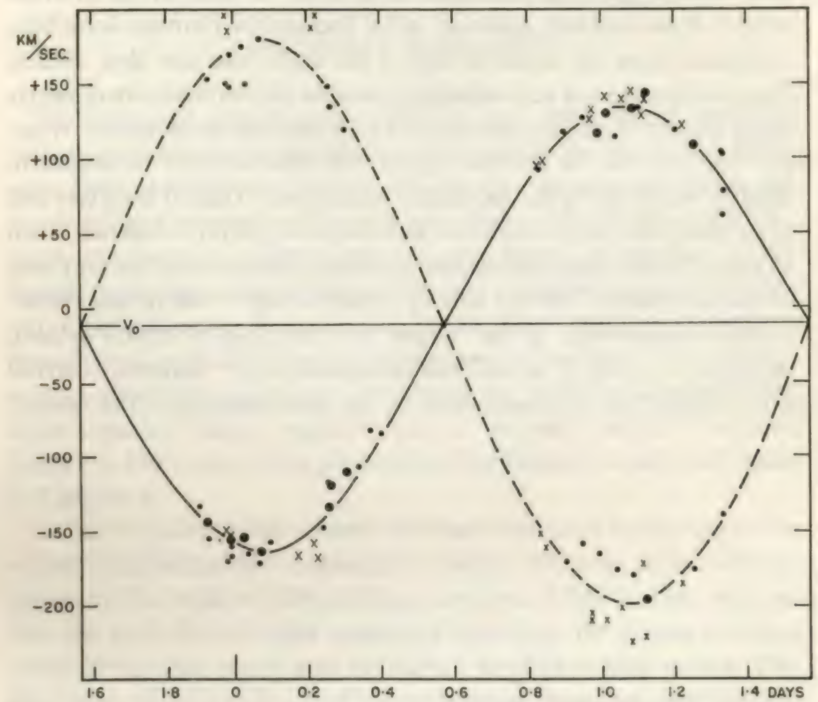

FIG. 1.5. Velocity curve for the double-lined spectroscopic binary H. D. 175544 (Thackeray and Tatum, 1966).

These changes make suspect any determinations of orbital elements based on observations of an eclipsing binary at only one epoch, and the high accuracy sometimes claimed for elements from such observations seems particularly doubtful. In addition, some light curves, such as those of systems exhibiting shallow partial eclipses, or of systems containing an irregularly variable component, are in principle virtually indeterminate. A wide range of assumptions about limb-darkening

and the ratio of their radii can be made for such systems without violating the observations.

It might be expected that visual binaries are relatively free from these difficulties of interpretation, since the two stars can be resolved, and their relative positions measured. Difficulties do arise in the determination of the elements, however, often because the elements have been computed from too small an arc of the orbit (van den Bos, 1962b). There is sometimes a real ambiguity because the two stars are of nearly equal brightness and at certain times they become unresolvable. When the two stars can be resolved again, the observer may be unable to identify which star is the secondary component. Thus at least two sets of elements may be derived, in which even the period is different, both of which fit the observations equally well. In such cases, the only way to decide between the two sets of elements may be to obtain radial-velocity observations of the system. For all types of binary system, therefore, it is well to be cautious before accepting "elements" derived from a single set of observations as the true elements of the system.

CHAPTER 2

THE INCIDENCE OF BINARY SYSTEMS

GENERAL REMARKS

There are two aspects to the study of any group of astronomical objects. One may either study the individuals in the group and determine their properties accurately, or one may be concerned with the distribution of members of the group throughout the Galaxy, or within some other selected group of stars. In this respect, astronomy is very similar to biology, since biologists also are interested in the properties of both individuals and species. In Chapter 1 the objects of study have been defined, and the necessary terminology introduced. This chapter, and to some extent the next, are concerned with the characteristics of the species "binary system" (and in the next chapter "multiple system"). The study of the properties of individuals is taken up again in Chapter 4.

In recent years, much interest has been apparent in the questions of the distribution and frequency of binary stars, as is particularly shown by the work of Abt and his associates. It has become obvious that the evolution of some groups of stars (e.g. the novae, and less certainly the Am stars) may be related to their binary nature. The discussion of the incidence of binary systems, therefore, has implications in the whole field of astrophysics. The study of the origin of binary systems may also be helped by accurate knowledge of their distribution in the Galaxy. There are two difficulties in these statistical investigations. The first is that great care has to be taken to eliminate the effects of observational selection from the results before firm conclusions are drawn. These effects may not always be easy to estimate quantitatively. The second problem, related to the first, is that it is not easy to prove that a given star is *not* a binary. When a claim is made that *all* members of a group of stars are binary systems, this is

almost impossible to refute, because a few instances of apparently single stars in a group are to be expected, either because of the orientation of the orbital planes, or because the properties of the hypothetical companion are such that it escapes detection. Thus, observationally it may be difficult or impossible to distinguish between the two propositions: *all members of this group of stars are binaries*, and *this group of stars contains a very high proportion of binaries*, yet the difference may be very important for the understanding of the origin of the group of stars. For these reasons, it is important to distinguish between the *detected* frequency of binary stars and their *true* frequency. The problem of determining the true frequency, given the detected frequency, is simply that of making the correct allowances for observational selection. These differ according to the method of observation used. There is, of course, an additional problem that the detected frequency of binaries among any group of objects can only be determined from a finite sample, and there is no guarantee, even when full and correct allowances have been made for selection effects, that the frequency found for a specific sample accurately reflects that in the entire population. It is assumed here that accurate inferences are possible, provided that the samples are reasonably large.

This chapter is concerned with the following specific problems: the incidence of binaries among main-sequence stars, amongst normal, Am, and Ap stars, in galactic clusters, amongst novae, variable stars, and Wolf–Rayet stars, and their relative incidence in Populations I and II. First, however, the effects of selection on the discovery of visual, spectroscopic, and eclipsing binaries are discussed.

SELECTION EFFECTS IN THE DISCOVERY OF BINARY SYSTEMS

Selection affects the discovery of visual, spectroscopic, and eclipsing binaries differently. Although the last-named group is a subset of the spectroscopic binaries, there are some situations (globular clusters, other galaxies) in which eclipsing binaries are the only ones that can be detected. It is, therefore, important to discuss the selection factors affecting their discovery, as well as those affecting the discovery of visual and spectroscopic binaries.

Distance is the most important selection factor in the discovery of *visual* binaries. Those closer to the Sun are more easily discovered than are those farther away, because the angular separation of the two components is greater for any given spatial separation. The detected frequency of visual binaries among stars of different spectral classes is therefore heavily biased by the distribution of nearby stars among these classes. Hot, luminous stars of early spectral type are rare in the solar neighbourhood, and a large portion of the known visual binaries (especially those for which orbits have been determined, because they are among the nearest) have components of late spectral type. Thus, allowance must be made for the different mean distances of various types of star before the statistics of visual binaries amongst those types are compared. Another selection factor is the magnitude difference between the two component stars. A faint companion is harder to detect than one nearly equal in brightness to the primary star, especially if the angular separation is small. An intrinsically luminous star is more likely to have a much fainter companion than is a faint star (if very low-mass companions and planetary bodies are ignored) so this selection factor also operates against the discovery of visual binaries amongst the luminous stars. The size of the factor can be determined if the luminosity distribution of stars is known, and if the relative difficulty of discovering a pair of stars of given magnitude difference and angular separation can be estimated. This has been done by Heintz (1969a).

Very distant companions create a further difficulty in the discovery of visual binaries, although this is perhaps not strictly a selection effect. Such companions are less likely to form a genuine binary system with the primary star, being rather, optical companions. To avoid including a large number of these in the statistics of binary stars, a particular spatial separation of a pair of stars is somewhat arbitrarily defined as the maximum possible for a real binary system. Some genuine binaries are then undoubtedly omitted from the statistics, while some optical pairs are incorrectly included. If the limit is carefully chosen,

however, estimates of the total number of binaries are probably not greatly in error. From the assumed upper limit to the real separation of the stars, a statistical relation between the angular separation and the apparent magnitude of the binary system can be derived and used to test the binary nature of any pair of stars. The problem has been discussed by Dommanget (1967).

The discovery of *spectroscopic* binaries is easier the larger their velocity variation. For a given orbital period and inclination, the larger velocity variations are found in the more massive systems. The detected frequency of spectroscopic binaries, therefore, is expected to be higher amongst the massive stars. Since these are usually hot, luminous stars, the *detected* frequency of spectroscopic binaries should show a reverse trend with spectral class from that of visual binaries. All estimates of the frequency of spectroscopic binaries amongst different types of stars must be reduced to equal mass before they are compared with each other. Thus, the detected frequency of spectroscopic binaries amongst, say, M-type dwarfs must be multiplied by the factor

$$\left\{\frac{\text{Mean mass of B-type stars}}{\text{Mean mass of M-type dwarfs}}\right\}^{1/3}$$

(because the orbital velocity is proportional to the cube-root of the mass) before being compared with the detected frequency amongst B-type stars. Although the selection depends only on the cube root of the masses, the factor can be as large as 2 or 3. There are other factors to consider. Binary systems containing stars of late spectral types tend, on average, to have longer periods than those containing stars of early spectral types. For a given total mass, the velocity variation of a spectroscopic binary is inversely proportional to the period. Thus the velocity variation of a typical late-type spectroscopic binary is even smaller than would be expected from its mass. This apparent trend in periods may itself be a result of selection, because a third factor is the accuracy with which velocity measurements can be made. The velocity of a star of late spectral type, whose spectrum contains many sharp lines, can usually be measured more accurately than that of a star of early spectral type, whose spectrum contains few, diffuse

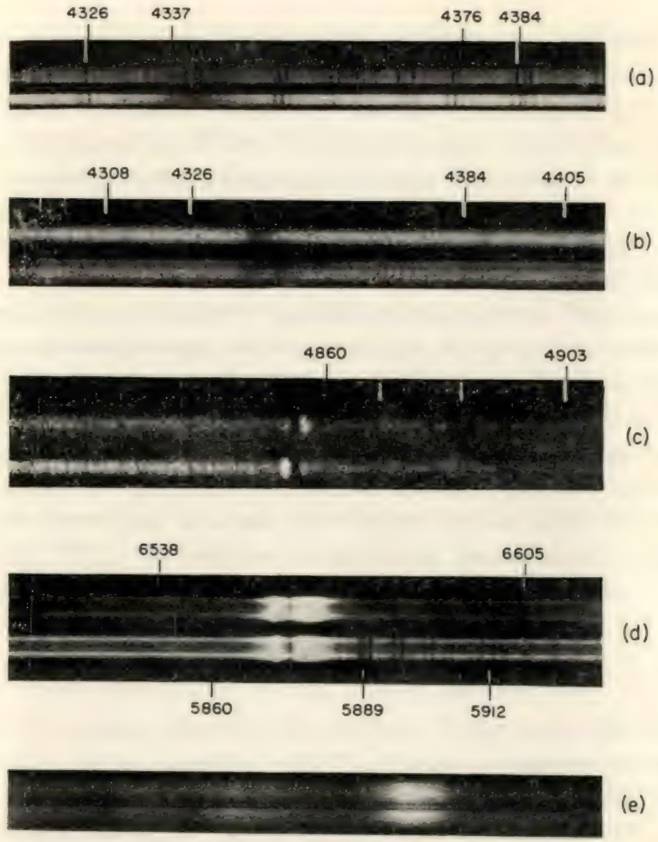

PLATE I. (a) Spectrograms of 47 Andromedae showing the region of Hγ at single-line (upper) and double-line (lower) phases. The spectrum is of Am type and the sharp lines are very easy to measure. (Original dispersion: 5 Å/mm.) (b) Spectrograms of H.D. 35921 also showing single (upper) and double Hγ lines. The spectrum is early B, and well illustrates the difficulties encountered in measuring this type of spectrum. (Original dispersion: 15 Å/mm.) (c) Spectrograms of SX Cassiopeiae shortly before (upper) and after (lower) primary minimum. The region shown is Hβ. Note that the strong emission component is to the red on the upper plate, and to the violet on the lower. (Original dispersion: 30 Å/mm.) (d) Emission in the spectrum of β Lyrae at Hα (upper) and λ 5876 of HeI (including the interstellar D lines of sodium). The phase is shortly after primary minimum. (Original dispersion: 10 Å/mm.) (e) Portion of the spectrum of the Wolf–Rayet star CQ Cephei showing the broad emission near λ 4686 and illustrating the difficulty of measuring such spectra. (Original dispersion: 15 Å/mm.)

lines. More low-amplitude (long-period) velocity variations, therefore, are likely to be observed amongst stars of late spectral types (Plates I(a) and I(b)). This factor will tend to reduce the discrepancy between detected frequencies of spectroscopic binaries in different spectral classes.

The probability of discovery of spectroscopic binaries depends on their orbital elements. This has been discussed in some detail by Scott (1951) in a publication that is unfortunately not readily available to many astronomers. She studied the effects of different values of e and ω on the probability of discovery of spectroscopic binaries, in order to find out whether or not selection effects can account for the non-uniform distribution of the element ω discussed in the last chapter. She found that systems with ω close to 0° or 180° are harder to discover than others, particularly if e is large, and K is comparable in size to the uncertainty of individual velocity measurements. Many of her results have been independently confirmed in unpublished work by Batten and Ovenden. Although this selection cannot explain the observed excess of systems with ω near 0°, it does indicate that the total number of spectroscopic binaries in any group of stars is likely to be underestimated, because some systems will have orbital elements that make detection difficult. The underestimate is more important for stars of early spectral type, because of the lower accuracy with which their velocities are usually measured. One-spectrum and two-spectra binaries have different probabilities of discovery because the first are detected by their velocity variations, and the second by their line doubling. The effects of e and ω on the discovery probability are similar for both types.

The discovery of an *eclipsing* binary depends on the fraction of time that its brightness is appreciably less than its maximum brightness (that is, both on the orbital period, and the fraction of the period that the eclipse lasts), and on the depth of the eclipse. The apparent brightness of the system is also important, especially for those systems discovered by photographic photometry, since variables near the plate limit are likely to be overlooked. Plavec (1968) has considered the relative probabilities of discovery of different kinds of eclipsing binaries of given characteristics. He calculated first the *geometric* proba-

bility of eclipse, which is simply the probability that a given system will be so oriented as to display eclipses, and then the *photometric* probability that these eclipses will be observable, which depends on their depth and duration, and is determined by the relative radii and luminosities of the two stars. In principle, the probability of discovering eclipsing binaries in any group of stars could be obtained by multiplying these two probabilities together for each individual binary system, and summing the product over all binaries in the group. In practice, however, the number and properties of binaries in the group are not known in advance, and a simpler approach must be tried.

Kvíz (1956) devised an approximate graphical method for deriving the probability of discovery of any variable star with a given light curve. It is illustrated in Fig. 2.1. His basic assumption was that the

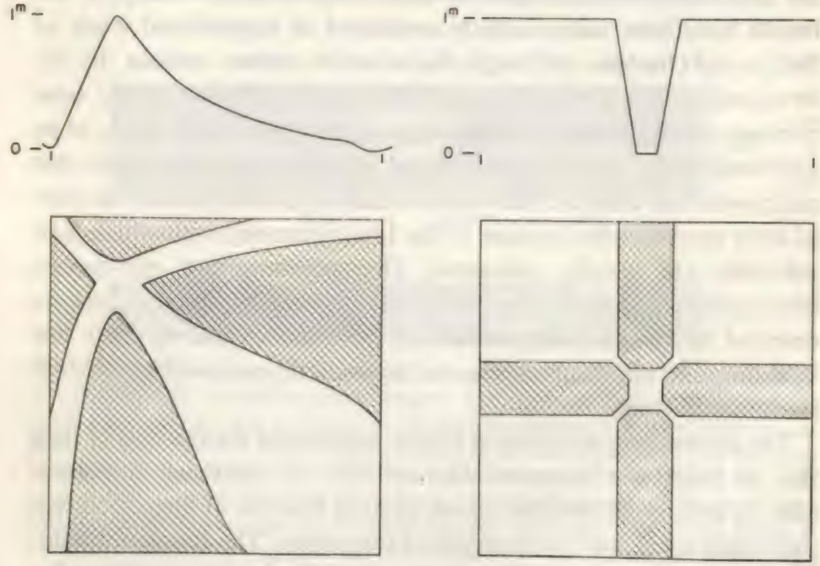

FIG. 2.1. Kvíz' method for estimating the relative probability of discovery of a variable star from the shape of its light curve. Schematic light curves are shown for an RR Lyrae variable (left) and an eclipsing variable. The ratio of the shaded areas within each closed rectangle to the area of the whole rectangle is a measure of the probability that any two observations of the light of the star will yield results differing by at least $0^m.25$.

variable was to be discovered by the use of a blink microscope, and would be detected if any two observations differed by more than a given magnitude difference Δm. The probability of discovery if the first observation is made at some given phase is the fraction of the period during which the magnitude of the variable differs by more than Δm from the magnitude it had at that phase. The probability of discovery from observations at any two arbitrary phases is the normalized sum of probabilities for all possible initial phases. Kvíz constructed this sum geometrically by plotting all possible initial phases on the y-axis against all possible phases of the second observation on the x-axis, and measuring the total area of regions in the resulting rectangle in which the magnitude difference exceeded Δm. In Fig. 2.1, such diagrams are shown for a schematic light curve of an RR Lyrae variable and an eclipsing variable. Each are assumed to have the same period and amplitude (1^m), and Δm is assumed to be $0^m.25$. The eclipsing variable has a probability of discovery 0.28, and the RR Lyrae variable has a probability 0.54. The eclipse is supposed to last for 20 per cent of the period, which is a fairly long duration. The secondary eclipse has been ignored, but this is not as important as it might seem. In the most favourable case of a binary displaying two equal eclipses, the probability of discovery is less than doubled by the extra eclipse. Moreover, the light of such a system cannot vary by more than $0^m.75$. An eclipsing binary containing two closely similar stars is harder to discover than some other types of system, because of this fairly low maximum range, and of the greater probability that eclipses will be only partial. A system whose light curve resembles the schematic curve with a flat maximum, illustrated in Fig. 2.1, is harder to detect than one having a continuously varying light curve of the W Ursae Majoris or β Lyrae types. Kvíz' method is only approximate, but its approximations are not important to this discussion of the relative probabilities of discovery of different kinds of variable stars. An eclipsing binary is certainly less likely to be discovered than is an RR Lyrae variable of the same period, range, and apparent magnitude. The probabilities become more nearly equal if more pairs of plates are examined.

The period of an eclipsing binary can modify its probability of discovery. Two artificial cases serve to illustrate the effect. Consider two eclipsing binaries of different periods, whose eclipses have the same depths and last for equal fractions of the respective periods. If observations are made during an interval that is long compared with both periods, the two binaries have virtually identical chances of detection, because each suffers the same diminution of brightness for the same fraction of the time. If the periods are very different, however, and the interval of observations is comparable to the longer of them, the probabilities of discovery may also be very different for the two systems. An eclipsing binary with a period of 1000 days may escape discovery for 2 years or more, even if eclipses last for 200 days, while another with a period of about 5 days, and eclipses that last 1 day, is very likely to be discovered during a 2-year interval. The long period decreases the chance of discovery. Consider also two other systems, each displaying central eclipses, and similar in all respects except the major semi-axes of their orbits. Let the sum of the radii of the two stars be R, and the semi-axes be a_1 and a_2. Then the durations of the eclipses are specified by the angles ϕ_1 and ϕ_2, where $\sin \phi_i = R/a_i$. The relative probabilities of detecting the two systems is ϕ_1/ϕ_2. For the values assumed by ϕ in most eclipsing systems, $\phi \approx \sin \phi$ is a fairly good approximation, so the probability of discovery is inversely proportional to the major axis, that is, it is proportional to $P^{-2/3}$. An eclipsing binary with a period of 1000 days has about one-hundredth the chance of discovery of one composed of similar stars with a period of one day. The known long-period systems often contain a giant or supergiant star, and the geometric probability that they will show eclipses is thus higher than it is for systems composed of main-sequence stars having the same separation. The first example shows, however, that even systems containing two giant stars have a low probability of discovery if the period is long. Most known systems like ζ Aurigae, 31, and 32 Cygni have low photometric probabilities of discovery; they were not discovered photometrically, but from their composite spectra. All lines of argument indicate that long-period eclipsing systems have a low chance of photometric discovery.

INCIDENCE OF BINARY SYSTEMS AMONGST MAIN-SEQUENCE STARS

Main-sequence stars can reasonably be regarded as normal stars, and, therefore, the frequency of binaries amongst them can be considered as a norm with which the frequencies of binaries in other groups of stars can be compared. This section is concerned with the frequency of all kinds of binaries amongst the main-sequence stars, but it is especially concerned with the frequency of spectroscopic binaries, because they are easier to detect in many other groups of stars, and their incidence can be more readily compared between these groups. In passing, the section also deals with the frequency of spectroscopic binaries amongst the giant stars.

The determination of the frequency of binaries amongst the main-sequence stars is beset by many of the difficulties discussed in the previous section, because these stars show wide ranges of mass, luminosity, and radius. One approach is to confine attention to a fairly small volume of space near the Sun, in which it may be hoped that nearly all the stars are well observed, and statistics are therefore complete. Any volume small enough to fulfil this condition contains very few stars at the luminous end of the main-sequence. Investigations of this kind have been made by Kuiper (1942) and van de Kamp (1969). Kuiper's investigation extends to about 10.5 parsecs from the Sun, and van de Kamp's to about 5 parsecs. The latter sample is very probably complete, but it contains no stars more luminous than Sirius (A1). Kuiper's sample contains no stars of any earlier spectral type, and although good statistics could be compiled out to somewhat greater distances, there are few stars of spectral type B within even 50 parsecs of the Sun. Van de Kamp's list contains fifty-nine stars grouped into thirty-one single, eleven (visual) double, and two triple systems. Seven invisible companions have been inferred from perturbations in the motions, only one of which is associated with a binary system that has already been counted. At least one of them is probably a planetary body, and they are left out of account here. Thus, among

forty-four objects, thirteen (or about 30 per cent) are at least double. There are no known spectroscopic binaries in this sample, but it consists mainly of dwarfs of K and M spectral types that are too faint for good radial velocities to be obtained. Kuiper's sample contains 109 components distributed between forty-four single stars, twenty-three double systems (including a few spectroscopic binaries, mostly of long period), five triple systems, and one quadruple system. That is, out of seventy-three objects, twenty-nine (or 40 per cent) are at least double. Since both these samples are small, the agreement between them is satisfactory.

Different investigators have used different methods of counting the duplicity in a sample of stars. In the above sample, Kuiper estimated the duplicity as 50 per cent, by which he meant that any randomly selected subset of, say, ten systems would contain, on average, fifteen components. He counted each *pair* of stars, so each triple system counted as two binaries, and the quadruple system as three. It is not true, however, that 50 per cent of the objects in the sample are multiple: only 40 per cent are. This method of counting exaggerates the number of multiple objects. The important fraction is the number of systems in a sample that are at least double. Corresponding fractions can be estimated for higher orders of multiplicity.

A different approach to the problem is to take a large sample of stars down to some given magnitude limit, and to correct for all the recognized selection effects. Kuiper (1935) did this for all stars down to $6^m.5$, and found a duplicity of 80 per cent by his method of counting. A precise conversion to the counting system preferred here is not possible because not enough details of the result have been published. Heintz (1967, 1969a) made a still more thorough survey of 8000 stars down to ninth magnitude. He used Kuiper's method of counting, but expressed it somewhat differently. He found that 100 objects chosen at random from these stars should contain between 200 and 210 stellar components, and he expressed this as a "duplicity factor" of between 1.0 and 1.1. Kuiper would have said that the duplicity is 100 to 110 per cent—but not all the stars in the sample are binaries. According to Heintz, 30 per cent of them are not. In this sample, 70 per cent of all stars are at least double. Heintz included spectroscopic binaries in his count by assuming the results of Jaschek and Jaschek (1957) described below. He confirmed the existence of a tendency (found by Worley, 1962) for there to be fewer very wide visual pairs amongst the dwarfs of late spectral types. Apart from this, the binary frequency appears to be roughly constant along the main sequence.

Other discussions of binary frequency amongst main-sequence stars have been concerned with *spectroscopic* binaries only. Jaschek and Jaschek (1957) considered all stars listed in the *General Catalogue of Radial Velocities* (Wilson, 1952) and in several other extensive published lists of radial velocities. They corrected for the different masses of stars of different spectral types, as discussed in the previous section, and found a roughly constant fraction of spectroscopic binaries (between 20 and 30 per cent) along the main sequence. The percentage of binaries amongst giant stars appeared to be distinctly lower, perhaps as low as 10 per cent.

Petrie (1960) tried to estimate the percentage of spectroscopic binaries in another way. He analysed the errors of radial-velocity measurements as reported in three extensive homogeneous programmes. He found that the frequency distribution of observational errors did not conform to the normal law, but showed an excessive number of very large errors (Fig. 2.2) which he attributed to the otherwise unrecognized variable radial velocities of many of the stars. He supposed that the observed error distribution was made up of two portions: the true observational errors, which he assumed to follow a normal law defined by the portion of the observed distribution to the left of the maximum; and real variations in velocity. If this is so, the percentage of variable-velocity stars is remarkably constant with spectral type at just over 50 per cent. Although spectroscopic binaries are not the only stars with variable velocities, Petrie felt that this figure gave a fair indication of the percentage of spectroscopic binaries, since certainly a number of stars that have variable velocities appear to have constant velocities because of the fortuitous timing of the observations. The two simplifications made in the analysis thus tend to cancel each other.

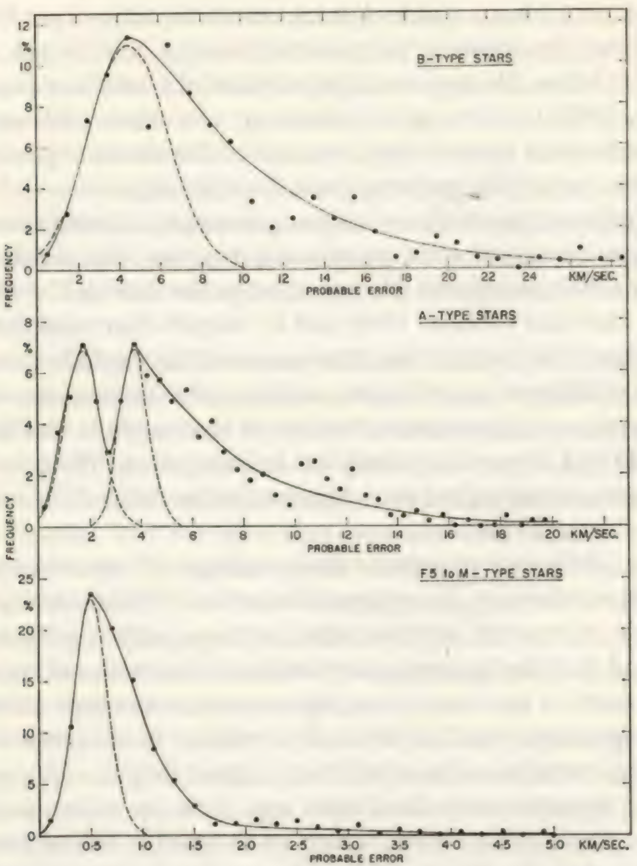

FIG. 2.2. Frequency distribution of dispersion of individual radial-velocity measures as found for samples of stars of different spectral types by Petrie.

Petrie's results have been criticized by Kirillova and Pavlovskaya (1963) on the grounds that the actual distribution of observational errors in a large sample such as he used is not expected to be a normal distribution. Jaschek and Gomez (1970) reconsidered the question, using much the same material, and assuming a theoretically more correct error distribution. Like Petrie, they find a constant percentage of spectroscopic binaries along the main sequence, although their

figure—47 per cent—is a little less than his. They also considered the percentage of spectroscopic binaries amongst late-type giant stars, and found it to be 30 per cent. Thus they confirm the result of Jaschek and Jaschek (1957) that the detected frequency of binaries is lower for the giant stars than it is for main-sequence stars, although their new figure is higher than that obtained by Jaschek and Jaschek. Because of the selection effects caused by the orbital elements, that were discussed by Scott, the true percentage of spectroscopic binaries amongst both groups of stars may well be somewhat higher than these figures indicate.

The samples used by Petrie contained very few known W Ursae Majoris systems. It is not quite clear whether or not such systems should be counted as containing main-sequence stars. Their components do not obey the normal mass-luminosity relation, and in the Hertzsprung–Russell diagram cluster only loosely about the main sequence. Shapley (1948) estimated that these systems were one of the most common types of variable, and Kraft (1967) estimates that one star in 2000 is such a system. It probably makes little difference, therefore, whether systems of this kind are included in the samples used in these surveys or not.

Apart from the possible decrease of visual binaries amongst the main-sequence stars of the very latest spectral types, most investigations seem to agree that the percentage of binaries among main-sequence stars is independent of spectral type. One exception is a study by O. C. Wilson (1966) who estimated the number of spectroscopic binaries in the Strömgren–Perry Catalogue, and found some evidence for a decrease in frequency of spectroscopic binaries with advancing spectral type. This was a by-product of the main investigation, however, and Wilson was careful to point out that he had made no allowance for selection effects.

The best estimate for the true frequency of binaries of all kinds, amongst main-sequence stars, seems to be Heintz' estimate that 70 per cent of all objects are at least double. Since he used the 1957 results of Jaschek and Jaschek for the frequency of spectroscopic binaries, however, the true frequency may be even higher. The best estimate

for the true frequency of spectroscopic binaries in the same group seems to be that it is approximately 50 per cent. This figure will be taken as the "normal" frequency of spectroscopic binaries throughout this chapter. The percentage of all objects among nearby stars that are visually at least double is about 40 per cent. Many of these stars are dwarfs of spectral types K and M, however, and in view of the result obtained by Worley and Heintz, that visual duplicity is lower than average amongst these spectral types, the true frequency of visual duplicity amongst all main-sequence stars is likely to be higher. Spectroscopic binaries seem to occur with about the same frequency amongst stars that are members of visual binaries, as amongst those that are not. All these figures may well be uncertain by ± 10 per cent, or more.

The apparently lower duplicity amongst giant stars may be genuine. Jaschek and Jaschek have pointed out that many binary components can never evolve into giants. There may also be a selection effect. Scarfe (1970) has found that single-spectrum spectroscopic binaries with giant components preferentially have mass ratios that differ considerably from unity. They are thus harder to detect, because the brighter components of such systems have only small ranges of velocity, and the fainter components are invisible. Scarfe believes, however, that a search with adequate spectrographic dispersion may reveal more two-spectra giant pairs.

INCIDENCE OF SPECTROSCOPIC BINARY SYSTEMS AMONGST A-TYPE STARS

In Fig. 2.2 the frequency distribution of velocity dispersions for the A-type stars has a double maximum. Petrie suggested that this is because of the division of stars of this spectral type into two groups: one of slowly rotating stars, and the other of rapidly rotating stars. Amongst the former group are found some stars with unusual spectra that are classified either as peculiar A-type stars (Ap) or as "metallic-line" stars (Am). In the Hertzsprung–Russell diagram they lie somewhat above the main sequence as defined by stars that have normal

spectra of similar type. There is some evidence that the frequencies of spectroscopic binaries amongst the Am and Ap stars are abnormal. This has recently become of special interest, because according to the modern theory of evolution of binary systems by mass exchange between the components, abnormal binary frequencies and spectroscopic peculiarities may be related. This is discussed in more detail in Chapter 10; in this section only the frequencies of spectroscopic binaries in these groups are discussed.

Petrie's work on the frequency of spectroscopic binaries amongst the A-type stars, discussed in the previous section, was based on a sample of 510 stars of types A and F observed by Harper (1937). The velocities were measured, and the spectra classified, before either Am or Ap stars were recognized as separate groups. Some stars in this sample are doubtless members of these groups, but Petrie's result refers simply to all stars whose spectra can be classified as "A". Jaschek and Gomez, in their discussion of Petrie's work, found that exclusion of the known Am and Ap stars from the sample makes little difference to the final result. This suggests that the frequency of spectroscopic binaries amongst stars with normal A-type spectra is normal.

Almost as soon as the Am stars were recognized as a distinct group, however, the frequency of spectroscopic binaries amongst them appeared to be higher than normal (Roman *et al.*, 1948). This is difficult to be sure of, because of the uncertainty about the normal frequency. Abt (1961) selected twenty-five known Am stars at random, and studied their binary characteristics. He found that twenty-two are binaries, although possibly four of these are not certain*. The orbital planes of some binaries must be oriented so that the radial velocities are small and escape detection, and Abt concluded that very probably all stars in his sample, and therefore all Am stars, are spectroscopic binaries. Strictly, however, the most that can be said is that 72 per cent of the Am stars in Abt's sample are known to be

* In an earlier discussion (1967a), I suggested that five of these stars were uncertain binaries. The duplicity of one of them, ξ Cephei, has since been confirmed by observations obtained by C. D. Scarfe in the fall of 1970.

spectroscopic binaries, and more may be. The frequency of spectroscopic binaries is certainly higher than normal in this sample of Am stars, and is probably high in the whole class of these stars.

In a complementary investigation (1965) Abt took a similar random sample of fifty-five normal A-type stars and studied the frequency of spectroscopic binaries amongst them. Surprisingly, he found no binaries with periods of less than 100 days, and only seventeen binaries or suspected binaries altogether. (Doubt has since been cast on the binary nature of one of these—μ^1 Bootis—by Niehaus and Scarfe, 1970.) The detected frequency of spectroscopic binaries in this sample is, therefore, about 30 per cent. If the discovery of spectroscopic binaries in this sample is complete, the frequency of such binaries amongst normal A-type stars is probably lower than normal, and certainly lower than that found for the small sample of Am stars. The first part of this statement contradicts the result obtained by Jaschek and Gomez. The absence of short-period binaries among the A-type stars was puzzling. Many short-period A-type binaries have been known for some time, and are listed in the orbit catalogues. Their spectra, however, were classified before the metallic-line spectra were recognized as a separate group, and many stars were described as of spectral type "A" which would not now be so described without qualification. Abt and Bidelman (1969) have reclassified a number of such spectra, and now recognize a small group of stars having more-or-less normal A-type spectra, and periods less than 2.5 days. Many of the stars previously classified as having A-type spectra, however, are now considered to show at least incipient "metallic-line" characteristics in their spectra. There is still, according to Abt and Bidelman, an apparent dearth of normal A-type binaries with periods between 2.5 and 100 days.

Abt has interpreted these results as evidence that there is an inverse correlation between the mean rotational velocity of stars in a group, and the frequency of spectroscopic binaries in that group. As I have pointed out (1967a), there is an obvious selection effect favouring the discovery of spectroscopic binaries amongst Am stars, because the spectra of these stars have many, sharp lines and small velocity

variations can easily be detected. On the other hand, the spectra of normal A stars often show few lines (principally the hydrogen lines which are intrinsically broad) broadened by rotation. A discrepancy in the detected frequencies of spectroscopic binaries, in the sense found, is therefore, to be expected. It is uncertain whether or not this selection effect can account for the whole of the observed difference. The absence of short-period systems amongst Abt's sample of normal A-type stars is evidence against the selection effect being the sole factor, because these systems, having in general large velocity ranges, ought to be least affected by such selection. On the other hand, there is also evidence that any correlation is in the opposite sense from that proposed by Abt. For instance, in Fig. 2.2 itself, the tail of large "observational errors" is attached to the second maximum for the A-type stars, implying that the stars with variable velocity are to be found among the rapidly rotating objects, and not the slowly rotating ones. Some confirmation of this is found by Jaschek and Gomez, who also conclude the frequency of spectroscopic binaries is *higher* among the rapidly rotating stars. Van den Heuvel (1968) and Deutsch (1967) obtained similar results for the B and A stars; they identified a group of slowly rotating stars of this spectral class, for which the detected frequency of spectroscopic binaries is lower than it is in the class of rapidly rotating stars of similar spectral type. Thus there is a contradiction between the results of different methods of studying this problem, which has not yet been fully resolved. The possible correlation between rotational velocity and binary frequency is discussed again in the next section, in the light of results about binary frequency in galactic clusters.

Abt has not yet published the results of any survey of the Ap stars parallel with his surveys of the A-type stars and the Am stars. Preliminary results indicate that the detected frequency of spectroscopic binaries amongst the Ap stars is certainly not high (private communication) perhaps around 25 per cent. Jaschek and Jaschek (1958) have studied the frequency of spectroscopic binaries in this group, and found it to be low (less than 20 per cent). Aikman (1971) tentatively finds about 50 per cent of binaries among Ap stars of

the mercury–manganese type. All the samples are small, however, and the differences between them may not be significant. The Ap spectra are also characterized by sharp lines, and velocity variations are fairly easy to detect. The most obvious selection effect therefore, should favour the discovery of spectroscopic binaries in this group, just as it does for the Am stars. Unless some other unsuspected selection factor is operating, it seems likely that the apparent deficit of spectroscopic binaries amongst Ap stars is real. These stars form another group in which slow rotation appears to be combined with low binary frequency.

INCIDENCE OF SPECTROSCOPIC BINARIES IN GALACTIC CLUSTERS

Although visual binaries are readily detected in nearby clusters, they are not so easily found in the more distant ones, and comparative studies of the binary frequencies in galactic clusters are best restricted to spectroscopic binaries. It is sometimes stated that close binaries are absent from galactic clusters. This statement seems to have been based on experience with the Pleiades, in which there are indeed few known spectroscopic binaries. The orbit of only one two-spectra binary in that cluster has been determined, and that was discovered quite recently (Abt, 1958; Pearce, 1957). Abt and his colleagues have studied the frequency of spectroscopic binaries in galactic clusters (Abt *et al.*, 1965; Abt and Hunter, 1962; Abt and Snowden, 1964; Chaffee and Abt, 1966; Geary and Abt, 1970; Abt *et al.*, 1970). Other investigations have been made by other groups, and Blaauw and van Albada have studied the incidence of both visual and spectroscopic binaries in a number of associations (1968). The results are summarized in Table 2. The lower figures given for each cluster are the percentages of known binaries in the samples studied in each group; the upper figures are the percentages of known and suspected binaries.

The detected frequencies of spectroscopic binaries are different in the different clusters. Abt has used these results to support his proposed correlation between binary frequency and rotational velocity,

TABLE 2. FREQUENCIES OF SPECTROSCOPIC BINARIES AND MEAN ROTATIONAL VELOCITIES IN TWELVE GALACTIC CLUSTERS AND ASSOCIATIONS

Cluster	Percentage of spectroscopic binaries (known—suspected)	Percentage of S.B.'s among stars with $v \sin i$		Mean rotational velocity (km/sec)	References
		<50 km/sec	>50 km/sec		
α Persei	11.0–35.6[1]	14.3	40.7	163	Kraft (1967)
Coma	20.5–32.3	28.6	38.4[6]	56	Kraft (1965)
Hyades	17.4–31.9[2]	27.4	47.1	53	Kraft (1965)
I.C. 4665	"many"	—	—	140	Chaffee and Abt (1967)
N.G.C. 6475	42.1–47.4	40	45.4	93	Abt *et al.* (1970)
Pleiades	12.3–24.6[3]	45.4	19.6	144	Anderson *et al.* (1966)
Praesepe	19.7–28.5[4]	36.3	33.3	98	Dickens *et al.* (1968)
Ursa Major	16.3	30	6.9	114	Geary and Abt (1970)
I Lacerta	16.9–47[5]	—	—	136	Abt and Hunter (1962)
I Orion	16.9–47[5]	—	—	143	Abt and Hunter (1962)
II Perseus	16.9–47[5]	—	—	160	Treanor (1960)
Sco-Cen	9.8–11.0	5.2	12.7	176	Slettebak (1968)

[1] Independent estimate by Petrie and Heard (1970) 6.5–16.8%.
[2] Independent estimate by Treanor gives 26%.
[3] Independent estimate by Abt *et al.* (1965) gives 12.7%.
[4] Independent estimate by Treanor, from smaller sample, gives 8%.
[5] Binary frequency is mean for all three groups: estimated by Blaauw and van Albada (1963) from different samples from those used for rotation determination.
[6] Includes two binaries with unknown rotational velocities.

claiming that the low binary frequencies are found in the clusters in which the average rotational velocity is high. He has also suggested that tidal friction in the close binaries reduces the rotational velocities of the component stars (Abt, 1970). The mean rotational velocities in each cluster are listed in Table 2. These are arithmetic means of the rotational velocities of all the stars in each cluster, except the

two-spectra binaries, the Am, and the Ap stars. Rotational velocities and binary frequencies are determined for the same samples of cluster members, except in the case of the three associations I Lacerta, I Orion, and II Perseus. The binary frequencies for these groups are the mean value found for all three groups (and the Cassiopeia–Taurus group) by Blaauw and van Albada (1963). The data of Table 2 are plotted in Fig. 2.3. Any correlation is obviously very weak. No cluster has a

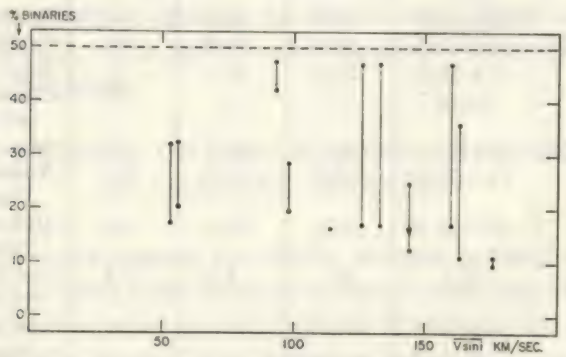

FIG. 2.3. Plot of percentage of cluster members that are known or suspected to be spectroscopic binaries against mean rotational velocity. Lower points for each cluster are percentages of known binaries, upper points of suspected binaries (see Table 2). Dashed line indicates the "normal" frequency of spectroscopic binaries.

high binary frequency if "high" is taken to mean higher than the normal main-sequence value*. The maximum frequencies found in the three associations—the only groups for which the investigators claim completeness of binary discovery—and in the cluster N.G.C. 6475 are very close to the normal frequency. These two facts strongly suggest that the greatest factor in the apparent difference of spectroscopic binary frequency from cluster to cluster is incompleteness of discovery.

The best way to compare binary frequencies in different clusters is to compare them amongst stars in each cluster that have similar

* Abt (private communication) has recently found evidence for supposing a binary frequency of 80 per cent in the cluster I. C. 4665.

rotational velocities. Any possible selection introduced by rotation is then eliminated. Unfortunately, sampling errors are likely to become important, because some clusters contain only very few stars having rotational velocities in a given interval (Abt, 1970). Nevertheless, columns 3 and 4 in Table 2 give the percentage of known and suspected binaries amongst slowly rotating ($v \sin i < 50$ km/sec) and rapidly rotating members ($v \sin i > 50$ km/sec) of the various clusters. Further investigation of the suspected binaries in these clusters is highly desirable. The figures in these two columns suggest that the chief factor inhibiting the discovery of binaries is not necessarily rotation of the component stars. The results from the three associations show that if a serious attempt is made to discover all the spectroscopic binaries in a group of stars, a very large proportion of them will be found, even if the mean rotational velocity of stars in the group is high. Large rotational velocities do create considerable doubt about the binary nature of individual stars; the difference between the numbers of known and suspected binaries seems to be greater in those clusters in which rotational velocities are high. The total number of variable-velocity stars known or suspected in these clusters, however, is not necessarily low. The apparent exception to this remark, the Scorpio–Centaurus group, perhaps illustrates it, because Slettebak's work was directed toward the determination of rotational velocities. He obtained only one spectrogram each for most of the stars, and most of the binaries he listed in his sample were known already. A new investigation, aimed at the detection of new binaries, might yield very different results. The Scorpio–Centaurus group of stars, and several other clusters listed in Table 2 contain more suspected binaries amongst their rapidly rotating members than amongst their slowly rotating ones. It is not clear that this is a true reflection of the actual numbers of binaries in these groups, but it is interesting that these clusters provide some support for the results obtained by van den Heuvel, and Jaschek and Gomez about the relative frequencies of binaries amongst slowly and rapidly rotating field stars. On the other hand, some clusters show the expected larger detected frequencies of binaries amongst their slowly rotating members, and one is tempted

to suppose that detection of binaries amongst the rapidly rotating stars in these clusters is seriously incomplete.

As evidence against the possibility that selection induced by rotation might be the cause of different binary frequencies in different clusters, Abt *et al.* (1970) have pointed out that in the Pleiades many of the binaries discovered have low amplitudes and that these should be the ones most completely concealed by rotational broadening of the spectral lines. (Pearce and Hill, 1971, question whether some of these stars really are binaries.) This evidence loses weight now that it appears that the majority of binaries detected in the Pleiades are slowly rotating stars. Perhaps in some clusters there is a preferred orientation of orbital planes, such that no high-amplitude spectroscopic binaries can be observed. Guthrie (1971) has also used Abt's results to point out the possibility that there is a preferred alignment of axes of rotation in some clusters. If there is a preferred orientation, there should also be some clusters in which high-amplitude spectroscopic binaries, or even eclipsing binaries, predominate. One example might be N.G.C. 6871, in which three of the seven brightest stars are spectroscopic binaries with known orbits, and two show eclipses.

Eclipsing binaries in galactic clusters are also important because they can help in the determination of the distance of the cluster. In turn, if the age of the cluster can be determined from the colour-magnitude diagram, a useful check on theories of evolution of close binaries can be made. Lists of eclipsing binaries that are, or possibly are, members of clusters have been published by Kholopov (1958), Kraft and Landolt (1959), and Sahade and Beron Davila (1963). The membership of some W Ursae Majoris systems in clusters has been discussed by Sahade and Frieboes (1960) and Eggen (1961). The rotational velocities of members of most of the clusters they list have not yet been determined, however, and the known eclipsing binaries in galactic clusters do not add very much to the present discussion. It is important, for many reasons, to investigate the membership of eclipsing binaries in galactic clusters more thoroughly.

BINARIES AMONGST CATACLYSMIC VARIABLES

Among the developments of the past few years, one of the most exciting is the realization that many, and possibly all, ex-novae are members of close binary systems. The astrophysical importance of this is discussed in another chapter: the concern in this chapter is once again with the observational evidence for the proposition that all novae are members of binary systems. Closely related to this proposition, and of course strengthening it, is the fact that the detected binary frequency among the U Geminorum stars is also high. The term cataclysmic variables has been used to describe novae, supernovae, and dwarf novae or U Geminorum variables. It is used here as a convenient generic term, although supernovae are not discussed because their binary nature must remain purely hypothetical. If they were binary systems, the supernova explosion disrupted the system.

The first ex-nova known to belong to a close binary system was DQ Herculis (Nova Herculis 1934) which Walker (1954) found to be an eclipsing variable. At that time the explosive variables SS Cygni and AE Aquarii were known to be binaries, and the recurrent nova T Coronae Borealis was suspected of being a binary. Struve (1955) suggested that perhaps all novae are binaries and that the binary membership is in some way related to the nova outburst. This concept has been elaborated by Kraft (1964b) who has also played the leading role in providing observational evidence to back it up (1964a, b). The evidence that all cataclysmic variables are members of binary systems can be simply stated. Forty-four per cent of all U Geminorum stars observable at Palomar have been shown to be very short-period binaries, and 37 per cent of all observable novae (70 per cent of those extensively observed) have been shown to be binaries, most of them with short periods, but with the notable exception of T Coronae Borealis (P = 228 days). In addition, there is evidence that two other ex-novae, V841 Ophiuchi and DI Lacertae, are seen pole-on, so 90 per cent of the ex-novae actually investigated either are, or may be, binaries.

Selection factors tend to work against discovery of binaries in this group of stars. All the stars are faint, and thus difficult to observe at all. Nearly all the periods are short, so if adequate time resolution is to be obtained, spectroscopically, a low dispersion must be used. This makes small velocity variations difficult to detect with certainty, a difficulty that is often aggravated by the nature of the spectrum. Absorption lines are weak and diffuse, if present at all; emission lines are sometimes sharper, but also often difficult to see and measure. Some old novae have spectra that appear to be featureless continua. Eclipses are of very short duration, often confused with other light variations, and sometimes very shallow. Therefore, both photometrically and spectroscopically, duplicity of these systems is very hard to detect. Indeed most of them have been detected only because deliberate and intensive searches have been made by Kraft, Krzeminski, and Mumford.

In view of these difficulties it seems almost certain that the true frequency of binaries among cataclysmic variables is higher than the detected frequency, and therefore probably very high, since the detected frequency in the observed sample is higher than the normal frequency. The sample size is small, however, and although the assumption that all cataclysmic variables are members of close binary systems has proved a fruitful working hypothesis, it cannot be regarded as observationally proved beyond all doubt. One definite case of a nova that is not a binary would be sufficient to throw doubt on the inevitable connection between binary nature and the nova phenomenon that is now often assumed. To prove conclusively that a given ex-nova is not a binary, however, would be very difficult, and the assumption that they are all binaries does seem to be a reasonable induction from the observations.

BINARY STARS AMONG ORDINARY VARIABLES

Struve and Huang (1957) suggested that the different kinds of stellar variability could be arranged in a continuous sequence at one end of which would be the regular, smooth variations of the Cepheid variables, and at the other end the novae. In the previous section, it was shown that many (and perhaps all) novae are members of binary systems, usually of short periods. Struve and Huang suggested that Cepheids are either single stars or components of very wide binaries. Abt (1959) found only 2 per cent of Cepheids are members of binary systems, and pointed out that Cepheids are not to be expected in systems with periods less than 100 days because an evolving O or B star in such a system would reach the critical surface imposed on it by its companion before it could expand sufficiently to become a Cepheid variable.

A more recent study of Cepheids has been undertaken by T. L. Evans (1968), and in a sample of about forty Cepheids of type I, he finds 15 per cent show some evidence of belonging to binary systems. Cepheid variables are giant or supergiant stars, and this percentage is not very different from the percentage of binaries found among giants by Jaschek and Jaschek, although it is distinctly below the corresponding figure given in the more recent work by Jaschek and Gomez. It is difficult to detect binaries among variable stars, however, because both the light and velocity variations to be expected from a close binary are distorted, or even hidden, by the intrinsic variations of the star. Many intrinsic variables are luminous stars observed at great distances: the detection of visual components is therefore also difficult. For example, it is still controversial whether or not the visual companion of δ Cephei is physically associated with that variable (Fernie, 1966; Worley, 1966). It thus seems likely that the true frequency of binaries among Cepheid variables is higher than the detected frequency, and that there is no strong evidence that it differs from the frequency found for the whole class of giant stars. Evans did find, however, that all the known orbital periods are considerably greater than 100 days. There is some evidence that there are fewer systems with periods in the neighbourhood of a year than there are with longer periods. The sample is not large enough for these conclusions to be other than tentative. Evans also found that Cepheids in binaries tend to have lower amplitudes for their intrinsic light variations than do other Cepheids.

A more complicated intrinsic variation is found in the β Cephei stars. At some phases their spectra show double lines, and for this reason many of them have been suspected of being binaries, although the line doubling is now known to have another cause. Their light variation usually shows two periodicities, and a beat period generated by them. Van Hoof (1965) has suggested that even more complicated multiple periodicities may be found for some of these stars. He believes that many of them are members of binary systems. Simon and Stothers (1969) have suggested that β Cephei stars are all members of close binary systems in which mass has been exchanged between the components. The outer layers of the now variable component contain material rich in helium from inside the initially more massive component. The mean molecular weight of these layers is thus higher than that of the whole star, and this difference is proposed as a mechanism to drive the pulsations of the star. The observational evidence for supposing all β Cephei stars to be members of close binary systems is ambiguous. Two of them are listed in the *Sixth Catalogue*, 16 Lacertae and σ Scorpii, although both orbits are of poor quality. It is difficult to separate the velocity variations due to orbital motion from those due to the intrinsic variation, and it is perhaps still not entirely certain that these stars are binaries. On the other hand, α Virginis —a known binary—displays a β Cephei light curve, as well as ellipsoidal light variations (Shobbrook *et al.*, 1969). Recently, orbital elements have been published for β Cephei itself (Fitch, 1969) and van Hoof has presented evidence that θ Ophiuchi is a binary (1967). The percentage of binaries among these variables is still unknown, however. There is again some evidence that companions, if they exist, modify the intrinsic variation. Van Hoof finds that the orbital period of σ Scorpii is close to an exact multiple of the beat period (1966), and Fitch has tried to explain periodic modulations of this type of variation as the effect of tides raised by the companion on the variable. Some binary systems are recognized in the related class of δ Scuti variables.

There is insufficient evidence available for a general discussion of the binary frequencies in other classes of variable stars. A few individual variable stars are known to belong to binary systems. For example, the long-period variable X Ophiuchi is a member of A.D.S. 11524 (Fernie, 1959).

WOLF–RAYET STARS

Another group of stars that has sometimes been supposed to consist entirely of binaries is the Wolf–Rayet stars. These stars are rare, about 120 are known in this Galaxy. According to Roberts (1962) about one-quarter of these are known or suspected to be close binaries although about half those brighter than $9^m.5$ fall into these categories (Underhill, 1966). Only ten appear in the *Sixth Catalogue* with determined orbits, and two others appear in the list of suspected binaries. Since selection effects are again important—many of the stars are faint, and the broad emission features in the spectra are hard to measure for radial velocity (see Plate I(e))—it has been suggested by many authors that all Wolf–Rayet stars are binaries. Opinion is divided on this point, and it might not be unfair to say that those whose chief interest is in binaries tend to be in favour of the hypothesis, while those whose chief interest is in the Wolf–Rayet stars tend to be against it. Underhill (1966) has suggested that the Wolf–Rayet stars could be a heterogeneous group containing members at different evolutionary stages. In this case, there may be a subgroup of the Wolf–Rayet stars (in a classification that is not yet fully understood) of which all members are binaries, and it may be that membership in a particular kind of binary system is necessary for the production of this kind of Wolf–Rayet star. Underhill has also argued (1959) that H.D. 192103 and H.D. 192163 are not members of binary systems. Absorption features are absent from their spectra and this rules out the presence of a companion brighter than absolute visual magnitude about -1^M (most of the recognized Wolf–Rayet binaries show two spectra, the second being that of an O- or B-type star). She also states that the radial velocities of these two stars are constant. This statement is based on only six and five spectrograms respectively, however, so the evidence is not conclusive.

The detected spectroscopic binary frequency amongst the brighter members of this group of stars is at least as high as the estimated normal frequency of spectroscopic binaries, and since the selection effects operating against the discovery of spectroscopic binaries in this group are strong, the true binary frequency may well be high. Whether or not it is 100 per cent remains an open question. Monteagudo and Sahade (1970) have drawn attention to variations in the emission intensities of the spectra of known Wolf–Rayet binaries that are dependent on the orbital phase. They suggest that these variations might be criteria of the binary nature of a Wolf-Rayet star that could be used even when the secondary spectrum, or radial velocity variations, are not detectable. This may provide a means of detecting Wolf–Rayet binaries that is more nearly free of the effects of selection.

RELATIVE FREQUENCIES OF BINARIES IN THE TWO STELLAR POPULATIONS

Important information about the origin of binary systems may be obtained from a study of their frequencies in stellar populations of different ages. If binary systems are formed by the simultaneous condensation of two stars around nearby separate nuclei, turbulence in the pre-stellar gas may be a factor that influences the number formed (Partridge, 1967). If the frequency of binary systems is very different in Populations I and II, this might be because of a fundamental difference in the conditions obtaining in the media from which the populations condensed. The information presently available does not justify consideration of possible variations in the incidence of binaries over such fine gradations in stellar population as were proposed at the 1957 Vatican conference (O'Connell, 1958). Therefore, only the classical division into Populations I and II is considered in this section. There are three ways in which the relative binary frequencies in these two populations can be studied. The first is to study the frequencies in samples of stars from each of the populations in our own Galaxy. The second is to study the frequency of binaries in globular clusters, which are pure Population II, and to compare this with the normal

frequency. The third is to study the binaries in a nearby galaxy. If it is not viewed edge on, the ratio of Population II objects to Population I objects should decrease from the centre outwards, and a difference in binary frequency between the two populations should show up, if it exists. By the last two methods, only the frequency of eclipsing binaries can be studied.

Partridge (1967) has found that the frequencies of visual and spectroscopic binaries are about the same in samples of the two Populations in our own Galaxy. His samples were the stars for which space velocities are listed in Gliese's *Catalogue of Stars Nearer than 20 Parsecs* (1957 edition). There are 600 such stars, and 213, by Partridge's criteria, are "high-velocity" (i.e. Population II) stars. He finds that 19.2 per cent of these, and 22.5 per cent of the low-velocity stars, are visual binaries. He also estimates 7.4 per cent and 6.6 per cent respectively for the detected frequencies of spectroscopic binaries. All these are much lower than the corresponding normal frequencies, and suggest that detection of binaries in these samples is seriously incomplete especially when the figures are compared with Kuiper's results for the volume within 10.5 parsecs of the Sun. Partridge has made little or no allowance for selection effects. Nevertheless, these are all nearby stars, and he finds no reason to suppose that selection effects act differently for the two populations. His results therefore indicate an approximate equality of the binary frequencies in these populations.

A contrasting view is presented by Abt and Levy (1969) who have investigated the number of spectroscopic binaries amongst five different groups of increasingly older stars and find no binaries at all of periods under 40 days in the oldest group, although 15 per cent of the stars in the youngest group are spectroscopic binaries with periods less than 100 days (median value 10 days). This sample is much smaller than that used by Partridge. Abt *et al.* (1970) have since found a short-period binary ($P = 6^d$) that is a metal-deficient star. It does not, however, share the kinematic characteristics of Population II stars.

Jaschek and Jaschek (1959) studied the frequency of binaries among the stars in Wilson's *General Catalogue of Radial Velocities*, having

velocities of qualities a, b, and c. Many of the suspected binaries in this sample are in clusters of known ages, and Jaschek and Jaschek found that the older clusters seemed to contain fewer binaries. They suggested that fewer binaries may have been formed in the older clusters, and also that evolution of stars in these clusters would tend to reduce the number of binaries detected. Components of binaries cannot evolve into objects that would belong to certain regions of the Hertzsprung–Russell diagram. Although this result does not apply directly to Population II stars, it may indicate a connection between the age of a group of stars and the frequency of binaries within the group. Thus, the results obtained from the study of stars in our own Galaxy are contradictory and inconclusive.

The detected frequency of eclipsing binaries in globular clusters is certainly low. Kopal (1959) stated that no certain member of a globular cluster was known to be an eclipsing binary. Recently, however, variable no. 78 in ω Centauri, which is an eclipsing variable, was shown to have a position, apparent magnitude, and proper motion consistent with membership in the cluster (Sistero, 1968; Geyer, 1967). A few other eclipsing variables are suspected in other clusters (Hogg, private communication) but some of the identified eclipsing variables in cluster fields are almost certainly foreground stars. Variable stars in globular clusters are usually found by the use of a blink microscope. It is shown in an earlier section that eclipsing variables are less likely to be discovered in this way than are RR Lyrae variables—the most common type of variable in globular cluster—of the same period and amplitude. Eclipsing variables in globular clusters are most unlikely to have periods and amplitudes similar to those of the RR Lyrae variables in the clusters. The latter usually have periods of less than a day; the former could have such short periods only if they were analogous to the W Ursae Majoris systems and composed of two faint stars nearly in contact. Such systems would be too faint to be detected in most globular clusters. Many of the RR Lyrae variables have amplitudes of two or three magnitudes. Eclipsing variables of Population I with such large amplitudes usually consist of a main-sequence star of late B spectral type accompanied by a late-type subgiant. In the older globular-cluster populations, these systems will have evolved further, and probably have become photometrically inconspicuous (Plavec, 1968a). If there are eclipsing variables among the brightest stars of a cluster—the red giants and supergiants—they are probably analogous to the ζ Aurigae stars, which, as is shown in the earlier section, are very difficult to detect photometrically. None of the known variables in ω Centauri has an amplitude of less than about $0^m.25$, and this is probably the limit of detectability. In this case, systems identical with 31 Cygni, 32 Cygni, or ζ Aurigae situated in ω Centauri could not be recognized as eclipsing variables. Actual binaries in globular clusters may be even harder to detect. Thus the probability of discovering such variables in globular clusters is very much less than that of discovering RR Lyrae variables. The discrepancy is increased by the diverse luminosities of stars in eclipsing binaries, while the RR Lyrae stars are all luminous. Most of the clusters that have been searched for variable stars contain very few—over half the clusters contain ten variables or fewer. About six clusters contain about 100 variables, or more, each. If the total populations of eclipsing variables and RR Lyrae stars are comparable in the clusters, then it is only in these few clusters that one can reasonably expect to discover eclipsing binaries at all. One of the clusters is ω Centauri.

Kopal (1959) estimated that about 0.2 per cent of the stars in the solar neighbourhood are eclipsing variables. If this figure is accepted as being typical of Population I, it is of interest to investigate the consequences of assuming that it is also typical of Population II. Globular clusters are usually supposed to contain between 10^4 and 10^5 stars (Hogg, 1959). They should therefore contain between twenty and 200 eclipsing variables. Many of these, however, are likely to be undetectable because the eclipses are shallow, the periods are too long, the systems are too faint, or they are situated in an inaccessible part of the cluster. It is probably not pessimistic to suppose that only about 10 per cent of these eclipsing binaries could be detected. A cluster like ω Centauri, therefore, should contain between two and twenty detectable eclipsing variables. The actual discovery of one is not far below expectations, and the apparent deficit of these variables in globu-

lar clusters is not sufficient to rule out the possibility that the frequencies of close binaries in the two main stellar populations are similar.

In other galaxies, as in globular clusters, binaries can only be detected photometrically. An important series of papers on variables in the galaxy M 31 has been published by Baade and Swope (1955, 1963, 1965) and Gaposchkin (1962). Although the prime purpose of these investigations was the study of Cepheid and RR Lyrae variables in that galaxy, a useful by-product was the discovery of a number of eclipsing variables. Four fields situated at 15, 35, 50, and 96 minutes of arc (south preceding) from the centre of M 31 are studied in these papers. The approximate true distances of the fields from the centre of their galaxy are 3000, 7000, 10,000, and 19,000 parsecs respectively, so the proportion of Population I objects to those of Population II should increase from field I (3000 parsecs) to field IV (19,000 parsecs).

TABLE 3. ECLIPSING BINARIES IN SELECTED FIELDS OF M31

Field	Approximate distance from centre of Galaxy	No. of eclipsing vars.	Approximate percentage of all vars.
I	3000	2	~ 2
II	7000	17	8
III	10,000	36	10
IV	19,000	10	18

The fraction of eclipsing variables (expressed as a percentage of all variables) and their absolute number do increase from field I to field IV (see Table 3) and this might be interpreted as prima facie evidence of a deficit of eclipsing binaries in Population II. Baade and Swope point out, however, that field I is very difficult to work in, because of the large amount of obscuring matter found there, and statistics for this field are probably incomplete. Furthermore, the same selection effects that have been discussed for globular clusters must apply here; the brightest stars of Population II are those among which eclipsing binaries are hardest to discover. If the evidence of Kraft and

his colleagues, discussed in an earlier section, that all, or nearly all, novae in our Galaxy are binaries is accepted, and if it is assumed that this is also true of novae in other galaxies, then seven novae must be added in field I, while only two are to be added in field IV. The increase of binaries outwards is then no longer so noticeable, except in field IV which is near to the edge of M 31, and may contain some stars that really belong to our own Galaxy. Finally, since the RR Lyrae stars are Population II objects, their number is bound to increase inwards, and the proportion of eclipsing variables to other variables will decrease inwards, even if the proportion of all stars that are eclipsing variables is constant throughout a galaxy. The evidence from M 31, therefore, does not unequivocally favour the view that there is a deficit of close binaries among stars of Population II.

None of the foregoing discussions is conclusive. The binary frequency of Population II stars remains an open question. No existing evidence clearly indicates that the true frequency of close binaries is any lower in Population II than it is in Population I. The results of this chapter are summarized in Table 4.

TABLE 4. FREQUENCY OF SPECTROSCOPIC BINARIES IN VARIOUS STELLAR GROUPS

Group	Frequency
Main-sequence stars	Normal
Giant stars	Apparently low
Am stars	High or very high
Ap stars	Low?
Galactic clusters	Normal (may differ in different clusters)
Cataclysmic variables	Probably very high
Cepheid variables	Probably low
Other variables	Unknown
Wolf–Rayet stars	Probably high
Population II stars	Unknown

"Normal" means 50 per cent of stars in group are spectroscopic binaries. The normal frequency of visual binaries is probably about 40 per cent. The variation of this frequency between these groups has not been studied.

CHAPTER 3

MULTIPLE STAR SYSTEMS

INCIDENCE OF MULTIPLE SYSTEMS

The study of multiple systems is a fascinating field that has been little explored. In 1935, in the second edition of his book *The Binary Stars*, R. G. Aitken devoted only a few paragraphs to selected multiple systems of particular interest. Not many workers have investigated such systems since. The first matter for discussion is the incidence of multiple systems. The difficulties discussed in connection with the determination of the incidence of binary stars are all to be encountered in this new problem. Indeed, their influence is likely to be greater. A third component of a system is usually separated from the principal pair by a greater amount than are the components of the pair from each other; thus, in a visual system, it is very difficult to establish a physical connection between all three stars. If all the components are detected spectroscopically, the orbital velocities of at least some of the components are likely to be small, and their variations hard to detect. The problems become greater the higher the degree of multiplicity of the system. In addition, in very complex systems it is hard to be sure that all the components have been recognized. Observational selection probably effects the discovery of multiple systems in a number of subtle ways. For example, Evans (1968) has pointed out that the *detected* frequency of spectroscopic multiples is much higher among stars with spectra of type F than among stars of other spectral types. He points out, however, that this is the optimum spectral range for the discovery of such systems. In spectra of earlier types the lines are usually too broad for the resolution of multiple components, and in spectra of later types there are so many lines that components of one spectrum are almost certain to be blended with oppositely displaced components of another. Whether these selection effects are

sufficient to account for the apparent excess of multiple systems with components of spectral type F, or whether there is a real variation of the frequency of multiple systems along the main sequence is not yet known. This chapter is not concerned only with spectroscopic multiples, however; full co-operation between spectroscopic and visual observers is needed in the study of multiple systems. Often the close pair of a triple is a spectroscopic pair, while the third body is a visual companion. Thus, Algol consists of an eclipsing binary with a period of about 2.8 days, attended by a third body with a period of 1.8 years. Although the triple nature of the system was originally discovered spectroscopically, an astrometric orbit has been obtained for the long-period system (van de Kamp *et al.*, 1950).

A number of attempts to estimate the frequency of multiple systems has been made: despite the difficulties of identifying multiple systems, their results agree fairly well. Petrie and Batten (1965) considered a sample of 234 visual binaries for which spectroscopic observations had been made at Victoria. These are mostly very wide binaries for which spectra of the two components could be obtained separately, so it was difficult to be sure that all the pairs of stars are true binaries. Evidence of either common proper motion, or even orbital motion, was available for many of the pairs, however, and for the remainder it was known that the radial velocities of the two components differed by less than 10 km/sec. This limit was set rather arbitrarily to allow for errors of measurement, possible relative radial velocities of orbital origin, and for possible variations in the velocities of one or both components. In this sample, eighty-two systems were either known to be multiple, or contained at least one component suspected of having a variable radial velocity. No account was taken of extra visual companions. Thus approximately 35 per cent of the sample of binary systems were found to be at least triple.

In another investigation, the present writer (1967) took the 737 spectroscopic binaries listed in the *Sixth Catalogue* and counted the number known to be spectroscopic triples, or listed as having at least a second companion in the *Index Catalogue of Visual Double Stars*

(Jeffers *et al.*, 1963). In this sample, 224, or 30 per cent, of the systems are at least triple.

Both of these estimates are very approximate. Little attempt has been made to take account of any possible selection effects, and it is probable that a number of spurious multiples are included in each. Thus the first sample may well include many stars suspected of variable velocity that are not really binaries, and the second sample certainly includes some optical "companions" since only the most obvious of these were excluded from the count. Moreover, the two samples are not completely independent: about fifty systems are common to both. Nevertheless, the agreement between them is encouraging, especially since the possible sources of error are different in each, and suggests that the true frequency of multiple systems may well be in the neighbourhood of 30 per cent of the systems that are at least double.

Worley (1967) studied the 536 visual binaries listed in his *Catalog of Visual Binary Orbits* (1963). This sample is better investigated than either of those just discussed, and contains only a few members in common with either of them. Worley counted all companions, whether detected by visual, astrometric, or spectroscopic techniques. He found 118, or 22 per cent, of the systems are at least triple. He believes that even in this well-investigated sample the detection of multiple systems is incomplete. For example, he points out the difficulty of detecting the known third component of α Centauri if the system were at ten times its actual distance (that is, astronomically speaking, still nearby), and he also mentions that nearly all the known third components are companions of the brighter star of the original pair, rather than the fainter. There is no known reason for this apparent preference. Worley suggests that the true frequency of multiple stars among the binary stars may be as high as 30 to 50 per cent.

Heintz (1969) has also used the *Index Catalogue* to provide a sample for the investigation of the frequency of multiple systems. His sample consists of all systems with a combined apparent magnitude of $8^m.9$ or brighter. He has been very thorough in the elimination of optical pairs, and finds only 590 systems (about 15 per cent of the sample) to be multiple. He believes that only about half of the multiples have

been found, however, so he estimates that about one-third of all binary systems are multiple.

There may also be undetected triple systems among the spectroscopic binaries (Batten, 1968). If an unsuspected third body is present, and if its orbital period is long compared with the period of the close pair, but short compared with the average interval of time required to obtain sufficient observations to determine the orbital elements of the close pair, then those elements are subject to systematic distortion. This is because each individual observation of velocity is made up of contributions from the motions in each orbit of the system. In general, the velocity curve of the close pair will be distorted differently at different times, so a spectroscopic "binary" that is really a triple may appear to have different orbital elements at different epochs. The element K, for example, may appear to vary between the sum and the difference of individual values of K in each orbit. Some evidence for changes in the element K is presented in Chapter 1, where other possible causes for them are discussed.

All these investigations agree in indicating that the proportion of multiple systems among stars that are at least double is quite high, and probably between one-quarter and one-third. Although some of the investigations are open to the objections that insufficient precautions have been taken to avoid selection effects, or to avoid the inclusion of spurious triples, the nature of selection effects in the different samples is different, and the agreement between diverse methods of investigation suggests that the result is approximately correct.

The difficulties of specifying the proportion of triples that are more complex are still greater. Not only are the difficulties that have already been discussed aggravated, but, in addition, unless the original sample of binaries is made unrealistically large, the numbers of higher-multiple systems included are so small that large sampling errors are likely to affect the results. Several estimates agree, however, that between one-quarter and one-third of all triples are at least quadruple. Heintz and Worley have independently suggested that the same fraction of quadruples are quintuple, while Batten has suggested one-fifth for the latter ratio. Several estimates are collected together in

Table 5, as far as the quadruple systems. The investigations by Wallenquist (1944) and Wierzbinski (1964) have been criticized by Heintz (1969) on the grounds that they did not include sufficient precautions to eliminate optical companions. The increasing percentages of higher multiples found by Wierzbinski may be explained by this fact. The other investigations included in Table 5 have already been cited in this section.

TABLE 5. FRACTION, x_n, OF ALL SYSTEMS CONTAINING AT LEAST n STARS THAT CONTAIN AT LEAST $(n+1)$ STARS

n	Wallenquist	Wierzbinski	Petrie and Batten	Batten	Heintz	Worley
1	—	—	—	0.50	0.70	—
2	—	0.19	0.35	0.30	0.33	0.22[1]
3	0.22	0.28	0.23	0.33	(0.33)	0.21
4	0.30	0.40	—	(0.20)	—	(0.28)

[1] Lower limit: true value is probably between 0.3 and 0.5.

MULTIPLE SYSTEMS AS A CLUE TO THE ORIGIN OF BINARY SYSTEMS

The existence of large numbers of multiple systems is relevant to the discussion of the origin of binary systems, because it suggests that duplicity is not an isolated phenomenon, but only a special case of multiplicity. If the origin of binaries is fission of a single protostar, multiple systems might be expected to be rather rare. It may be profitable to present these ideas in a more formal way, although the numerical values about to be given can only be considered as provisional because of the uncertainties discussed in this and the preceding chapters.

Let the fraction of all systems containing at least n stars that contain at least $(n+1)$ stars be denoted by x_n (where $0 \leqslant x_n \leqslant 1$). Both spectroscopic and visual multiplicities are to be considered. Numerical estimates of x_n are given in Table 5 for $n \leqslant 4$. In an earlier paper (1966), I suggested that the fraction of all stars containing at least n components is proportional to $1/n!$, so $x_n = 1/(n+1)$. If all the estimates gathered in Table 5 are considered, it seems more likely that x_n has a nearly constant value between 0.25 and 0.33 when $n \geqslant 2$. In view of the discussion in Chapter 2, Heintz' value of $x_1 = 0.7$ seems to be the best available. Let $f(n)$ be defined as the fraction of all systems containing exactly n stars. [The importance of this function was first pointed out by Kuiper (1935) although he published no attempt to determine it.] It is easily shown that

$$f(n) = (1-x_n)x_{n-1}x_{n-2}, \ldots, x_2x_1,$$

and if $x_1 = 0.7$, $x_n = 0.25$, $n \geqslant 2$, then

$$f(1) = 0.3, \quad f(n) = 0.525\ (0.25)^{n-2}, \quad n \geqslant 2.$$

If these figures are taken, for the moment, as indicating the true proportions of systems of different degrees of multiplicity, they can be compared with the predictions of the hypothesis that all multiple systems result from condensations of protostars about nuclei distributed in a random fashion. Suppose that the Galaxy has been divided up into cells of equal size that contain on the average one star each. Let a multiple system be any cell containing more than one star. If multiple systems have formed in a random manner, their distribution should be a Poisson distribution

$$P(n) = \frac{n_0^n}{n!} \exp(-n_0),$$

where n_0 is the expected number of stars per cell, and is by definition unity. Therefore,

$$P(n) = 0.368/n!$$

The function $P(n)$ is not quite the same as the function $f(n)$, because $P(n)$ is the fraction of all cells containing at least n stars, and $f(n)$ is the fraction of all systems (occupied cells) containing at least n stars, and $P(n)$ must be multiplied by a factor that eliminates the empty cells, that is by

$$1/[1-P(0)] \approx 1.58.$$

Therefore the predicted value of $f(n)$ is

$$f(n) = 0.58/n!$$

Table 6 compares the theoretical values of $f(n)$ (based on this particular hypothesis of their origin of multiple systems) with "observed" values based on the adopted values $x_1 = 0.7$, $x_n = 0.25$, $n \geqslant 2$. There are considerably more multiple systems than a simple theory of random condensation predicts, and all or most of the excess is found among the binary systems. These figures are thus evidence that some binary systems are formed in some other way than random condensation, although, in view of the uncertainties involved, there is no need to invoke any other theory for the formation of multiple systems. The result depends heavily, however, on the validity of Heintz' value of 70 per cent for the total multiplicity rate (visual and spectroscopic systems) among main-sequence stars. The conclusion, therefore, can only be tentative.

The fraction of sextuple systems given by the "observed" relation is approximately in agreement with the number actually detected. Worley's *Catalog* of visual binary orbits, and the *Sixth Catalogue* of spectroscopic-binary orbits contain between them approximately 1200 stars (allowing for those common to both of them) and there are two known sextuple systems (α Geminorum and β^2 Tucanae). If, however, the statistics are incomplete (as is very likely) then there are more stars of this multiplicity than the Poisson distribution predicts. If

TABLE 6. THEORETICAL AND OBSERVED VALUES OF $f(n)$

n	Theoretical	Observed
1	0.58	0.3
2	0.29	0.525
3	0.10	0.131
4	0.024	(0.033)
5	0.005	(0.008)
6	0.001	(0.002)

$x_n = 0.33$ for $n \geqslant 2$, as is quite consistent with the observations, these conclusions are not substantially altered.

The function $f(n)$ is obviously decreasing for higher values of n as far as it can be determined. If this trend continued indefinitely, there could be no star clusters, because at fairly small values of n (around 20) the function $f(n)$ would become smaller than the reciprocal of the estimated number of stars in the Galaxy and not even one cluster should be observed. Since many clusters with many more than twenty members are known, $f(n)$ cannot decrease indefinitely with increasing n. It would be of interest, but very difficult, to determine the precise value of n at which the nature of $f(n)$ changed. This result suggests that multiple systems and star clusters are different kinds of object, and are not distinguished only by an arbitrarily chosen number of components. What the nature of the difference between the two groups of objects may be is unclear. It is uncertain, for instance, whether the stars in the Trapezium should be considered as forming a cluster or a multiple system.

Luyten (1967) has given some results for multiplicity among common proper motion groups. He finds triple systems are much less common among these loosely connected systems than has been found here for more tightly bound systems. This result is perhaps not unexpected.

CHARACTERISTICS OF MULTIPLE SYSTEMS

RATIO OF PERIODS

It is usually stated that a triple system consists of a close pair attended by a distant companion, and a quadruple system of either a close pair attended by two distant companions or two close pairs whose mutual distance is much greater than either of those between their components. Wallenquist (1944) found that triple systems of this preferred type are indeed in the overwhelming majority. The expected preference exists for quadruple systems, but is not so well marked. Heintz' caution about the inclusion of optical pairs in Wallenquist's sample should be remembered, however. Evans has introduced

the useful concept of "hierarchy" to describe this arrangement (1968). This depends on the use of a diagram rather like a family tree to describe such systems. A triple system containing one close pair and a distant companion is regarded as of hierarchy 2 (see Fig. 3.1). The two preferred forms of quadruple are respectively of hierarchy

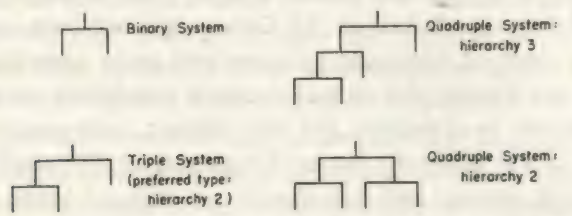

FIG. 3.1. Evans' concept of "hierarchy" of multiple systems.

3 and 2. If all the distances between components are comparable, the diagram representing the system becomes multiplex (more than two "descendants" in each "generation"). The solar system has a multiplex diagram, but systems containing stars with comparable masses are generally thought to be unstable if the mutual distances are also comparable. Although the question of the stability of multiple systems is not yet fully understood, the observed preferences seem to support the expectation that certain combinations are inherently unstable.

The large ratios in relative distances imply correspondingly large ratios in relative periods. The ratio of the two periods in the system of Algol is about 250, and this is often quoted as a typical value. There is a very wide range in the value of this ratio. Thus H.D. 100018 has periods of 7.4 days and 86 years—a ratio of about 4000—and A.D.S. 9537, each component of which is a W Ursae Majoris eclipsing binary, with a period of less than a day, is a common-proper-motion pair with a period so long that no orbital motion has yet been detected. (It is the only visual binary known consisting of two eclipsing binaries.) On the other hand, the triple system λ Tauri has an exceptionally small ratio of periods (about 8) and ζ Cancri (another quadruple) has its longest period only 20 times that of one of the component

visual pairs. Luyten (1967) has also described some faint, low-mass common-proper-motion triples in which the ratio of *separations* is 5 : 1, or less.

Figure 3.2 is a histogram showing the distribution of ratios of periods in multiple systems. The sample consists of all the multiple systems listed in the *Sixth Catalogue* and Worley's *Catalog* for which the ratio of periods is known. Some doubtful systems have been omitted, and some multiple systems have contributed more than one

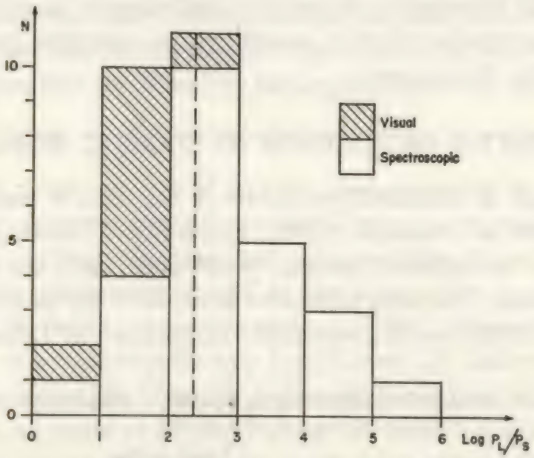

FIG. 3.2. Histogram showing observed frequency distribution of the ratio of long period (P_L) to short period (P_S) in triple systems discovered visually or spectroscopically. The dotted vertical line marks the value of the ratio for Algol.

value to the diagram. (The values taken are always the ratio of the longest period P_L to each of the shorter periods P_S in the system.) The sample is heavily biased by selection in favour of small values of the ratio P_L/P_S. This is most clearly shown by those systems whose triple nature has been discovered by visual means alone. Not only are there many fewer of these than of systems in which at least one of the orbits has been discovered spectroscopically, but all of them except one have ratios of periods less than 100 : 1. The exception is μ Draconis, in which the close pair is an unresolved astrometric binary. Close pairs discovered spectroscopically usually have periods

of only a few days, and thus much larger values of the ratio P_L/P_S are possible, in these systems, for a given value of P_L. It appears that Algol *is* fairly typical of those triple systems in which both periods have been determined. On the other hand, undetected triples, of the type discussed in the previous section, may have $P_L/P_S < 10^2$. Probably the statistics in all ranges of P_L/P_S are incomplete, and the true "typical" value is unknown. A perusal of Worley's *Catalog* reveals a surprising number of visual binaries known or suspected to contain spectroscopic binaries for which no orbital elements have been determined. Investigation of these would help to complete the statistics on which Fig. 3.2 is based.

RELATIVE INCLINATION OF ORBITAL PLANES

A problem of considerable interest is the relative inclination of orbital planes in a multiple system. Eggen (1962) has suggested that they tend to be coplanar, although Wallerstein (1963) has questioned this conclusion. The most exhaustive analysis so far was undertaken by Worley (1967) and his material is reproduced in Table 7, except for two systems for which the information is incomplete. This investigation was made some time after the publication of his *Catalog* and Worley was therefore able to discuss the orbits of a few more systems than are included in the discussion of the previous paragraphs. There are nine systems, giving ten values of relative inclinations, since one of the systems is quadruple. Worley concluded that there is a strong tendency for coplanarity of orbits of multiple systems. This conclusion was questioned by van Albada (1968), however, who pointed out that Worley had always chosen the numerically smaller of the two possible values of the relative inclination. The inclination, ϕ, between two orbital planes that are defined by the angles i_1, i_2 and Ω_1, Ω_2 is given by

$$\cos\phi = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos(\Omega_2 - \Omega_1).$$

If, however, either or both of Ω_1 and Ω_2 are uncertain by 180°, the + sign in the equation must be replaced by a ± sign, and two values of ϕ are possible. In the absence of radial-velocity observations, there is always an uncertainty of 180° in the value of Ω as mentioned in Chapter 1. In the list of systems given in Table 7, the ascending node is known without ambiguity only for ζ Cancri CD, and even then not for ζ Cancri AB-CD. Thus, for ζ Aquarii, the orbital planes may be inclined at an angle of 92°.2 (87°.8) or 137°.6 (42°.4) (Strand, 1941). The figures in parentheses are the supplements of the angles obtained from the formula, and can be used whenever the relative directions of motion in the two orbits are irrelevant to the discussion. Harrington (1968) has argued that triple systems containing mutually perpendicular orbits are very unstable, and therefore the value of 42°.4 is more likely for ζ Aquarii. Even this value, however, hardly indicates that the orbits are coplanar.

A closer examination of the values tabulated by Worley shows that not one of the ten values of ϕ is less than 10°, and only one is less than 20°. This is in spite of the consistent choice of the numerically smaller value of the inclination. Worley's conclusion that orbits in multiple systems tend to be coplanar seems to have been drawn from his statistical analysis of individual values, in which he has confused the probability that the angle ϕ should have at least a given value with

TABLE 7. ANGLES BETWEEN ORBITAL PLANES IN MULTIPLE SYSTEMS

System	Least angles between planes
−30° 529	34°.2
A.D.S. 1860	29.2
A.D.S. 3358	41.3
−50° 2240	(73.0)
A.D.S. 6650 (ζ Cnc ABCD)	29.7 43.5
A.D.S. 6993	38.4
A.D.S. 11635	19.6
A.D.S. 15971	(87.8)
A.D.S. 16185	(82.0)

Angles in parentheses are the supplements of the smaller of two possible obtuse angles.

the probability that that value, when actually observed, has arisen by chance. His individual results provide no clear evidence for this conclusion. His collection of the available data is, however, most useful and serves to indicate the uncertainty of the observational base on which any deductions in this field must necessarily rest.

RELATIVE SENSES OF REVOLUTION

Another characteristic of multiple systems that is of interest is whether the two orbital motions have the same or opposite senses. Although this question has a clear and precise meaning when the two orbits are coplanar (or nearly so), it is not so clear when the inclination between them is high. To make it clear, imagine tangents drawn to each orbit from the point of intersection of the orbits, in the direction of motion. If the angle between these tangents is acute, the systems are said to be co-revolving: if it is obtuse, they are counter-revolving. If the relative inclination is exactly 90° the situation is ambiguous, but this case may be unlikely on theoretical grounds. Heintz (1969) has pointed out that even if all systems are co-revolving, a certain percentage will appear to be counter-revolving because the Earth happens to be situated in the acute angle between the two orbital planes. Similarly, even if all systems are counter-revolving, some will appear to be co-revolving. If the relative inclinations of orbital planes in multiple systems are distributed at random, the probability of any given angle ϕ between orbital planes is given by

$$\sin \phi \, d\phi,$$

and the probability that the Earth will be situated inside the acute angle ϕ, rather than the obtuse angle $\pi - \phi$ is simply ϕ/π. The angle ϕ can assume any value between 0 and $\pi/2$, and therefore the total fraction of apparently counter-revolving systems in a pure population of co-revolving systems is given by

$$\int_0^{\pi/2} \frac{\phi \sin \phi \, d\phi}{\pi} = 1/\pi \simeq 0.31$$

Investigations of this problem have been undertaken by van den Bos (1928), Strand (1941), and Worley (1967). Worley's investigation is based on a total of 54 systems, and is again the most extensive made to date. Each of these systems shows enough evidence of orbital motion for relative senses of revolution in the two component orbits to be determined. Amongst the triple systems, Worley found that 78 per cent are apparently co-revolving; but amongst the multiple systems he found only 57 per cent co-revolving. The number of apparently counter-revolving systems among the triples is, therefore, approximately what would be expected (although, if anything, a little less) if all systems were in fact co-revolving. It is thus tempting to express the conclusion that co-revolution is the rule. Counter-revolving systems can appear to be co-revolving, however, and since the ascending nodes remain ambiguous in most of the systems in Worley's sample, the relative inclinations of the orbital planes and the true relative senses of revolution remain unknown. Some systems seem definitely to be counter-revolving, for instance ζ Aquarii, for which *either* possibility for the ascending node implies an obtuse angle between the orbital planes. It is not clear whether the seeming small deficit in the number of apparently counter-revolving systems, compared with the prediction, should be regarded as definitely established. It could be interpreted as an indication that not all values of sin ϕ are equally probable—that, for example, very high values of the inclination are avoided: they are the values most likely to cause a given system to appear counter-revolving. The higher percentage of apparently counter-revolving systems among the multiples may not be significant, as Worley says, because of the small sample size. It could be interpreted, if significant, as an indication that high inclinations between the orbits of multiple systems are more common than between the orbits of triple systems. This would create a larger proportion of both genuine and apparent counter-revolving systems.

SPECTROSCOPIC-VISUAL TRIPLES

An interesting class of triple systems is that in which one component of a visual binary is a spectroscopic binary. Very often, a visual binary that has been successfully resolved with the aid of a long-focus refractor appears as an unresolved single star on the slit-head at the Cassegrain or coudé focus of a reflector. This is particularly true of those binaries for which orbits have been determined, since these visual binaries are predominantly short-period pairs, that is to say pairs with small separations. This is one reason why so disappointingly little information is available on, for instance, the orientation of the orbital planes of visual binaries. The radial velocities of the components of most visual binaries are small, and if the binary happens to consist of two nearly equal components, the two component spectra may never be successfully resolved at any point in the cycle and no Doppler shift can be measured in the spectrum. Those few systems whose components show orbital motion *and* can be resolved sufficiently for their spectra to be obtained separately have even smaller velocity variations. They can be profitably observed spectroscopically at only the very highest dispersions, and, until recently, that has meant that only a few of the brightest systems can be observed. The situation is changed, however, if one of the components is a spectroscopic binary, for then a large, rapid, velocity variation is superimposed on the small, slow variation in the visual orbit. If the two components of the spectroscopic binary are also comparable in luminosity, then the spectroscopic observer may periodically see triple lines, the lines of the spectrum of the visual companion falling between the lines of the spectra of the other two components. Systems fulfilling all these conditions are probably rather rare, but a selection is known already. They include p Velorum (F4, F4, and A0?), H.D. 118261 (three F-type stars), and possibly H.D. 188088 (K5), all discovered by Evans (1968), H.D 100018 (A.D.S. 8189, F2, F2, F5—Petrie and Laidler, 1952; Petrie and Batten, 1970) and H.D. 165590 (A.D.S. 11060, three G-type stars) which has been under observation at Victoria for a long time, although the presence of three spectra in the combined

light has only recently been established. As mentioned in the first section of this chapter, Evans has pointed out that this kind of system is found predominantly among stars of spectral type F. This may be a selection effect.

In principle, the masses of all components in such systems may be completely determined, even though no eclipses are observed. The mass-ratio of the visual companion to the spectroscopic pair can be determined from the spectroscopic observations. The total mass can be determined either from the spectroscopic observations (if the value of i obtained from the visual orbit is used) or from the visual observations (provided the parallax of the system is known, as it will be if radial-velocity observations are available). Thus the quantities m_1+m_2 and m_3 can be obtained. The ratio of m_1 and m_2 can also be obtained from the spectroscopic observations, so the mass of each component can be determined. The inclination of the spectroscopic orbit to the tangent plane of the sky can also be determined, but not the relative inclination of the two orbital planes, since the node of the spectroscopic orbit with the plane of the sky remains unknown. The relative inclination, ϕ must lie between

$$i_s-i_v \leqslant \phi \leqslant i_s+i_v,$$

however, where i_s and i_v are the inclinations of the spectroscopic and visual orbits to the tangent plane respectively. The combination of the visual and spectroscopic orbits also leads to the determination of the parallax of the system. Thus the masses and luminosities of the three component stars can be completely determined, and such systems ought to be able to yield three very strong points for the empirical mass-luminosity relation. Alternatively, a colour-magnitude diagram can be constructed for the system (or, more correctly, a spectrum-magnitude diagram, since the individual colours are probably unobservable). It will have very few points, compared with a similar diagram for a cluster, and the magnitude differences, being spectrophotometrically determined, may be only approximate, but the mass corresponding to each point will be known.

Unfortunately, as is often the case, what is easy in principle is difficult in practice. One difficulty has been emphasized by Evans: if there are more than two components in the system, it is very difficult to be sure just how many there are, because the blending of the spectra of different components, inevitable in late-type spectra, may conceal some of them. The only practical method of procedure is trial and error. One measures all the suspected components, later rejecting those that are found to be present for only one or two lines on any plate, or those that do not make dynamical sense.

Even if the number of stars in a system can be clearly established, however, there are other difficulties. The relative radial velocity in the visual orbit is likely to be small, and therefore both the total mass $(m_1+m_2+m_3)$ and the ratio $m_3/(m_1+m_2)$ can be determined only with considerable uncertainty. This is aggravated by the fact that the velocity of the centre of mass of the spectroscopic pair has to be inferred from the measured velocities of the two components. Systematic errors may also be introduced by the blending of different components of the same line: the three line profiles may blend together in such a way that the positions of minimum intensity are displaced. This effect has been investigated by Petrie *et al.* (1967) and Batten and Fletcher (1971). It is discussed in more detail in Chapter 5, in connection with the determination of stellar masses. The effect appears to be unimportant if the members of the spectroscopic binary have large enough velocities for the spectral lines of all three stars to be separated by more than about 1.5 times their half-widths. That is, if the orbital velocities are greater than about 15 km/sec and a dispersion of about 10 Å/mm has been used, then this source of error should be unimportant for stars of late spectral types. The visual orbit may also be poorly determined, because many of these spectroscopic-visual triples are rather close visual systems, and therefore difficult to measure.

This situation is illustrated only too well by observations of H.D. 100018. Despite the small angular separation of the two visual components (maximum 0".5), it seems likely that the visual orbit, which now depends on observations covering nearly two of the 86-year periods, is fairly reliable. The primary is a spectroscopic binary with

a period of 7.4 days. All three stars have spectra of middle F type, and at the nodes of the spectroscopic orbit the three sets of lines are well enough separated for the blending errors to be negligible. Spectroscopic observations have been made of the system during the interval in which, according to the visual orbit, the relative velocity of the visual components should have reached its maximum. Figure 3.3 shows that it did indeed reach a maximum, but the observed value was about half the expected value. The discrepancy cannot be removed by adjusting the parallax to accord with the radial-velocity observations, because this introduces unacceptable values for the masses—and, indeed, a contradiction between the demands of the spectroscopic and visual orbits. It appears likely that there is no gross error in either the visual or the spectroscopic observations, but the accumulation of many observational difficulties in both methods of observation has produced this fairly large systematic error. Finsen has suggested (private communication) that a new method of orbit determination is needed for systems of this kind. At present, the usual practice does not make full use of the radial-velocity observations, but is an attempt to force them to fit the "best" orbital elements that have been derived by a least-squares procedure from the visual observations alone. Kerrich

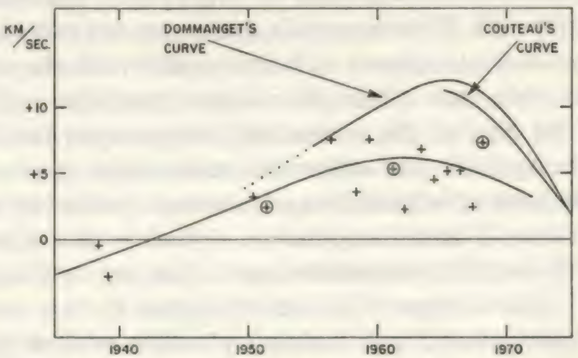

FIG. 3.3. The radial velocity of the third body in H.D. 100018, relative to the centre of mass of the spectroscopic pair. Circled points have the greatest weight. Predicted radial velocities are shown as computed by Dommanget and Nys (1967) from orbital elements by Muller (1955), and by Couteau (1965) from independently derived orbital elements.

(1930) suggested a different method, but this suffers from the opposite disadvantage that it tends to force photographic observations of a visual binary to fit elements derived from the spectroscopic observations. Unfortunately, if the component stars can be resolved at all, the spectroscopic elements are likely to be of low precision. A new method of orbit determination is needed that makes use of all the information contained in both kinds of observation. Its development presents an interesting mathematical problem, because the visual (or photographic) observations provide the two coordinates in the tangent plane of the sky, but the spectroscopic observations supply only the time derivative of the third coordinate. Such a method would be useful for all visual binaries that had also been observed spectroscopically, but it would be particularly useful for this group of spectroscopic-visual triples, which, although a small group, is potentially an important one.

The system p Velorum is an interesting member of this group. One component of the visual pair is a spectroscopic binary composed of two F-type stars. The other visual component is very hard to classify, because its spectrum can only be inferred from its effect on the hydrogen lines, the K line, and the line λ 4481 of Mg II in the composite spectrum. It was studied in detail by Evans (1956) who revised Sanford's (1918) orbit of the close pair, and concluded that the spectral type of the companion is early A. It is impossible to resolve completely the spectra of the stars in this system at any phase because of the great width of the lines of the, presumably, A-type spectrum. Only the combined magnitude and colour can be measured photometrically, so any discussion of individual magnitudes must proceed by successive approximations in such a way that no violence is done either to the measure of combined magnitude and colour, or to the estimated individual spectral types. This was attempted by Arp and Evans (1956) who concluded that the most plausible solution places the A-type star below the main sequence. Although this conclusion has been questioned by Eggen (1959), and is clearly not a unique one, it provides an example of the sort of result that may be obtained from the careful observation of triple systems.

CHAPTER 4

THE PERIOD OF A BINARY SYSTEM

THE DETERMINATION OF THE PERIOD

This chapter, and succeeding ones, are concerned more with the properties of individual binaries than with the properties of groups of binaries. One of the most fundamental properties of an individual system is its orbital period. Although it was defined as one of the elements of a system in Chapter 1, the period deserves a special treatment of its own; principally because it can often be determined with an accuracy that far surpasses that of the other orbital elements. The attainable accuracy in the period depends on the cumulative effect that any small error in its determination will have over a large number of cycles. The periods of spectroscopic and eclipsing binaries with short periods (up to a few days) can be determined very accurately. That of an eclipsing binary showing deep, well-defined eclipses that can be accurately timed can, in exceptionally favourable circumstances, be refined to an accuracy of one part in 10^9. Periods of spectroscopic binaries that do not eclipse are usually less well determined, because the continuous variation of velocity does not provide any clearly defined points such as the contacts of an eclipse, and the accuracy of radial velocities is usually less than that of photoelectric observations of the light of a system. The periods of long-period systems are not, of course, determined so accurately, although there is no reason in principle why eventually they should not be. The periods of visual binaries are rarely known to better than one part in 10^3 or 10^4. Few visual binaries have orbital periods less than 10 years, and long intervals of time must elapse before such periods can be determined accurately. Some of the longest periods quoted are based on only a small arc of the orbit, because a complete revolution has not yet been observed, and they may well prove to be wrong.

If a periodic phenomenon can be observed frequently at regular intervals, then its period is fairly easy to determine. Unfortunately, the astronomer is rarely in this position: he is usually prevented, either by the weather or by the scheduling of the telescope from quickly accumulating a long series of observations at short, equal intervals. The determination of the period of a new binary is something of an imprecise art, and it is easy to obtain a wrong value. All methods are ultimately based on trial and error. This is even true of the recently developed computer programs (Evans and Young, 1966; Lafler and Kinman, 1965; Deeming, 1970). Their chief advantage is that they greatly increase the number of trials that can be made, and similarly decrease the length of time needed to make the trials. Of course, they also increase the number of errors that can be made. The basic method is to pick out the extreme values of the light or velocity variation, and to assume that the smallest interval between extrema of the same sign is the orbital period. It may, of course, be a multiple of the period. This method is easier for some eclipsing binaries. The light variation caused by eclipses is limited to a fairly small portion of the orbital period, and there are definite discontinuities in the slope of the light curve that serve as markers. There are other eclipsing binaries in which neither of these conditions obtain: the two components are so close to each other, and distorted, that the variation of light between eclipses is sufficient to make the whole light curve appear smooth, and to hide the times of the beginning and end of eclipses. The velocity curve of a spectroscopic binary is always a continuous curve, the individual velocities that define it are known with much less accuracy than are good individual light measurements, and it cannot safely be assumed that the maxima and minima of velocity are separated by exactly half a period, as that is only true if the orbit is circular. After a number of possible values for the period have been selected, it is necessary to calculate the phase of every observation with the assumed value of the period, and then to plot the observations with this value of the period to see if they define a permissible light or velocity curve. Usually several such cycles of computation are needed. The computer program devised by Evans and Young not only calculates the phases, but displays the velocity curve on an oscilloscope screen, and thus enables the investigator to select quickly the most promising values for further study.

It is only too easy in this work to find a false period for a system. Each kind of binary, eclipsing, spectroscopic, and visual, is subject to its own special kinds of error in period determination. The simplest kind, inherent in the very method of period determination and therefore common to all kinds of binary, is that the period found will be a small multiple of the true period. Such an error is most difficult to detect in those eclipsing binaries that display light curves containing successive minima of equal depth. It is often difficult to decide whether these are successive eclipses of the same bright star by a dark companion whose own eclipse is barely detectable, or alternate eclipses of two nearly equally luminous stars. The period in the second case, of course, is twice what it would be in the first case. A good example of this sort of confusion was Y Cygni, which does contain two nearly equal early type stars, and for a long time the true period was uncertain (Dunér, 1892). The situation was further confused because the true period is almost exactly 3 days, so from any one observatory only alternate minima are observable over quite long time intervals. The difficulty is perhaps less than it used to be, now that modern photoelectric observations permit the detection of very shallow secondary eclipses, but it may still be present in the early stages of observation of a new binary. For example, H. D. 205372 was recently discovered to be an eclipsing binary by the Bamberg observers (Strohmeier *et al.*, 1963). They announced for it a period of half a day, but later spectroscopic observations showed the true period to be in the neighbourhood of a day (Mammano *et al.*, 1966). The photometric observers supposed that H.D. 205372 was an Algol system, with no perceptible secondary eclipse. The spectroscopic observers detected two spectra, thus showing that the system must contain two nearly equal stars. It is, in fact, an early-type contact system.

A more subtle kind of spurious period is sometimes found for spectroscopic binaries. This arises from the periodicity that may be imposed on the observations by the exigencies of the observing pro-

gramme. Although the astronomer cannot usually make observations at regular, short intervals, he is constrained to observe most stars at particular times of the year, and he usually tries to observe them when they are near the meridian. Thus a periodicity is imposed on the observations corresponding to the length of a sidereal day. If the true period is close to a sidereal day, or even a small whole number of days, then an entirely erroneous period may be determined. The effect is similar to that observed in the stroboscope (Fig. 4.1). It has long been known, and has been discussed by Hagen (1921) and Struve (1928). A more

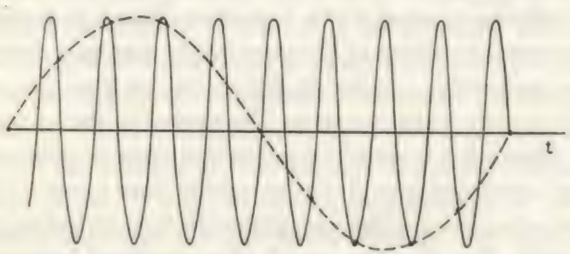

FIG. 4.1. A spectroscopic binary with a period near to one day, or an integral number of days (solid curve), can appear to have a much longer period if observations are made at intervals approximately equal to the true period. Velocity (V) is plotted against time (t) in arbitrary units.

recent and very useful discussion of it is that by Tanner (1948) who devised a test for distinguishing such spurious period, and applied it to the periods found for several binary systems (1949). There can be many spurious periods P' related to the true period P by the formula

$$\frac{1}{P'} = \frac{1}{P} \pm \frac{n}{kt},$$

where t is the least interval of observation (in this application a sidereal day), n is a number that indicates the approximate period in days ($n = 1, \frac{1}{2}, \frac{1}{3} \ldots$ corresponds to periods of $1, 2, 3 \ldots$ days approximately), k is an integer indicating the number of days, on average, between successive observations. In practice, the most common case is

$k = 1$, and probably also $n = 1$. The two spurious periods are then given by

$$\frac{1}{P'} = \frac{1}{P} \pm 1.0027,$$

where P' and P are expressed in mean solar days. It is quite important to test for these spurious periods before accepting a newly determined period as final. The simplest test is deliberately to make a number of observations when the star is well off the meridian, thus breaking the false, imposed periodicity. Unfortunately, it is not always possible to do so, as, for example, when one attempts to observe a star of southern declination from a northern observatory. In such cases, it may be impossible to eliminate the spurious periods unless observations have been made at more than one observatory. Trumpler (1930) published observations of a southern star, H.D. 159176, which can be equally well satisfied by periods of either 4.920 days or 3.368* days, and it is impossible to choose between these two values from Trumpler's observations alone. Tanner has shown that the presence of a correlation between the hour angle of an observation, and its residual *in phase* from the mean velocity curve is a good indicator that the chosen period is spurious. There should be no such correlation if the true period of the system has been found.

If the spectra of both components are visible, and they are closely similar, there may be further confusion because an error of a half period may be made in assigning a phase to a given observation. This is particularly likely if the orbit has a small eccentricity. If the eccentricity is large, the symmetry of the velocity curve may be destroyed and the separation of the two spectra will be quite different at opposite nodes.

An erroneous period may also be derived when a few early discovery observations are used in conjunction with a more numerous series to obtain the period. This is a very good way of refining a provisional

* Thackeray (private communication) states that new observations by T. L. Evans confirm the shorter period.

period, and it is often used because an observer frequently finds himself with just such a distribution of available observations. The comparison of two sets of observations made at very different times quickly reveals any error in a provisional value of the period derived from only one of them. There is a danger, however, that another error may be made in the number of cycles assumed to have elapsed between the early and the modern observations. Thus, although a spurious period may be eliminated by this procedure, a small residual error may escape unnoticed until further observations are available.

The period of a spectroscopic binary which has a large orbital eccentricity and a longitude of periastron close to 0° or 180° can be very hard to determine, especially if the period is long (some tens of days) and only a few cycles can be covered in an observing season. The observed velocity may be nearly constant over most of the period, while in a short part of the cycle the variation is rapid, and can easily be missed. If two spectra are visible they may only be resolved during the interval of rapid velocity variation. For example, 47 Andromedae was first supposed by Jose (1951) to have a period of 39.393 days, an orbital eccentricity of about 0.6, and a longitude of periastron close to 0°. More recent observations did not fit these elements, but eventually Fletcher (1967) showed that the true period is 35.371 days, or about 9/10 of the period found by Jose.

It might be thought that a visual binary whose apparent orbital ellipse is traced out against the sky in the course of a cycle would not present problems of period determination. Because the apparent orbit is a projection of the true orbit, however, the two stars may sometimes appear to oscillate past each other in a straight line, or at least they may be unresolvable for a portion of the cycle. Either of these is the more likely to happen if the true orbit has a large eccentricity. If the two components are of similar brightness and colour, there may be a very real doubt as to their identity when they can again be observed separately. In such cases, two completely different values of the period may appear equally probable. Properly timed radial-velocity observations can sometimes solve the problem. A specific example has been discussed by van den Bos (1962c).

ACCURACY OF PERIOD DETERMINATION

The remainder of this chapter is concerned with real and apparent changes in orbital periods. These periods can be determined more accurately for eclipsing binaries than they can for any other kind of system, and the rest of the chapter, therefore, is concerned with eclipsing binaries. In this section, the accuracy with which periods can be determined is investigated.

The period of an eclipsing binary is usually considered to be the interval between two successive instants of mid-eclipse of the same component. The accuracy with which it can be determined, therefore, depends on that with which individual times of minima are known. This depends on the method of observation, the depth and nature of the eclipse, and the accuracy of individual observations. The time of the middle of a deep, symmetrical, total eclipse, observed photoelectrically in good skies, can be determined to within about a minute, or perhaps even less. That is, the uncertainty of the time of mid-eclipse is between 10^{-3} and 10^{-4} of a day. Suppose that minima have been observed at times t_0 and t_N with uncertainties ε_0 and ε_N, and let t_N be separated from t_0 by N periods. Then,

$$NP = t_N - t_0 \pm (\varepsilon_0^2 + \varepsilon_N^2)^{1/2}.$$

If $\varepsilon_0 = \varepsilon_N = \varepsilon$, then,

$$NP = t_N - t_0 \pm \varepsilon\sqrt{2},$$

$$P = \frac{t_N - t_0}{N} \pm \frac{\varepsilon}{N}\sqrt{2}.$$

If $(n+1)$ different minima have been observed, and if $\bar{N}$ is the average value of N, and the value of ε is the same for all observed minima, then the uncertainty in P is approximately

$$\frac{\varepsilon}{\bar{N}}\sqrt{\frac{2}{n}}.$$

This result shows, as is intuitively obvious, that the accuracy with which a period can be determined is directly proportional to the aver-

age number of cycles between the first and the later observations, and thus, in principle, a period can be obtained to any desired degree of accuracy by observing enough minima over a long enough interval. In practice, an upper limit is set to the accuracy of our knowledge of periods because few eclipsing binaries have been systematically observed for longer than a century, and none at all for as long as two centuries. The periods of most eclipsing binaries are of the order of a day, and the best possible value of $\bar{N}$ is, therefore, about 10^4. For a well-observed system, n is probably of the order of 10^2, while, in the most favourable cases, ε is likely to be of the order of 10^{-3} or 10^{-4} of a day (and therefore of a period), as stated above. Thus the best accuracy currently attainable in the period is of the order of one part in 10^8 or 10^9. Most periods will be known much less accurately. The earliest times of minima were not determined photoelectrically, or even photographically, and some are based on visual *estimates* (rather than measures). In such cases, the assumption that $\varepsilon_0 = \varepsilon_N$ is not justified; rather ε_0 may be greater than ε_N by a factor of 10, or even of 100, and the accuracy of the period determination would be roughly proportional to ε_0. Other systems have been observed for much less than 10^4 cycles, and their periods are known less accurately. The accuracy with which most periods are known is probably about one part in 10^7, although the periods of newly discovered or infrequently observed systems or of those that display shallow eclipses are still less well known. The length of the period does not have as much effect on the accuracy as it might seem to at first sight, because the reduction in $\bar{N}$ to be expected in a long-period system is compensated by a corresponding reduction in ε (expressed as a fraction of the period). Fewer minima are probably observed for long-period systems, however, and for this reason their periods will be determined less accurately.

So far it has been implicitly assumed that the orbital period of any given system is constant, and can be identified with the interval of time between successive mid-eclipses of the same component. In some systems, however, this interval varies; often because of a changing geometrical aspect of the orbit, in which case the change in period is only apparent; but sometimes because the orbital period itself is variable.

The smallest detectable period change, real or apparent, is obviously determined by the accuracy with which the period is known. Thus it is important to determine as many times of minima as possible, by good modern photometry. This work will yield its full value in the decades, or even the centuries, to come. Our ideas of which systems are important may well differ from those of our successors. Therefore we should observe many different systems, and not only those that are of interest to us.

A change in period can be detected when it causes a delay or advance in the expected time of minimum that is significantly greater than the value of ε, the uncertainty with which an individual minimum can be timed. If the $(O-C)$ residuals from some assumed ephemeris are plotted against the number of cycles elapsed, they should cluster around the line $(O-C) = 0$, if the period chosen is correct. If it is too short, each minimum will be later than predicted, and each successive minimum will be later by a greater amount. The $(O-C)$ plot will be a straight line sloping towards the upper right in the $(O-C)$-time diagram. This is illustrated in Fig. 4.2, based on a diagram published by Plavec (1962) for the system RW Tauri. The period of this system seems to have decreased suddenly about 1921 although it may have begun to decrease as early as 1913; the diagram is plotted with the

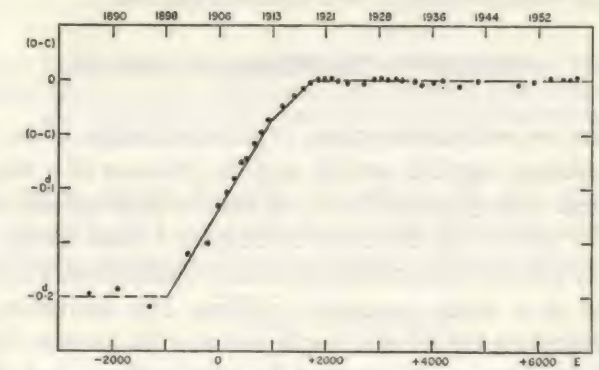

FIG. 4.2. Sudden change of period of RW Tauri as shown by residuals of times of minima. The points plotted are normal minima. See text for discussion. Dates in upper scale are approximate.

modern period. The minima observed between 1898 and 1921 clearly require a longer period than do those observed since. Obviously, if the points were clustered about a line sloping toward the lower right of the diagram, the assumed period would be too long. If the period of a system is constant, the $(O-C)$-time plot will always be a straight line: if the period changes continuously, the plot will be a curved line. The change in period of RW Tauri appears to have been abrupt, but there is always sufficient observational error to obscure the distinction between an abrupt change and a very rapid change that was continuous for a short time. Any period change, no matter how small, can eventually be detected if no further changes modify its effects. In practice, however, a limit is again set by the length of time for which eclipsing binaries have been observed. If the period is changed abruptly, the delay or advance in times of minima is linear in the number of cycles N, and the smallest detectable change will be of a similar magnitude to the accuracy with which the period is known; namely, in present circumstances, one part in 10^7 for most systems, and one part in 10^8 or 10^9 for exceptionally well-observed systems. If the period increases or decreases continuously, by a given amount in each cycle, the delay or advance in times of minima varies approximately as N^2, and much smaller changes in period—one part in 10^{11} per cycle, or less—can be detected for even moderately well observed systems.

APPARENT CHANGES OF PERIOD

There are two well-known causes of periodic changes of the interval between minima—apsidal motion, and the presence of a third body in the system. The dynamical theory of, and the observational evidence for, apsidal motion are discussed in Chapter 6. This section is concerned only with its effect on the apparent orbital period, which can be considered as a purely geometric problem. The derivation of the formula describing this effect is simple, and was first given by Tisserand (1895). It has been rarely repeated by other authors, and since the volume of *Comptes Rendus* in which Tisserand wrote is not readily available, it may be worthwhile to repeat the derivation here. Tisserand's own derivation has been expressed in more modern terminology, and some sections of it that are not strictly necessary have been omitted.

Suppose a binary system to consist of two spherical stars moving in an orbit whose plane contains the line of sight (i.e. the orbital inclination, i, is 90°). Let ω be the longitude of periastron of the orbit of the primary star relative to the secondary, and let v be the true anomaly of the star in this orbit (see Fig. 4.3). Let t be the observed time of mid-eclipse, and let the suffix "0" refer to the first eclipse observed, and the suffix "E" to an eclipse observed E periods later. (It follows that E must always be a positive integer.) Then an eclipse occurs when

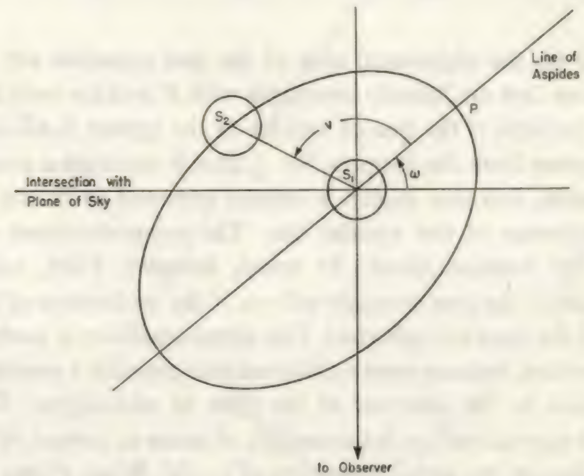

FIG. 4.3. Explanation of nomenclature used in the derivation of Tisserand's result. The orbit lies in the plane of the paper. The periastron point is labelled P, and S_1 and S_2 are the two stars. The angle PS_1S_2 is the true anomaly, v.

$$v+\omega = \pi/2+2\pi E. \tag{1}$$

Now, by a well-known formula in celestial mechanics:

$$M = v-2e \sin v+\text{higher terms in } e,$$

where M is the mean anomaly in the orbit, and therefore at the Eth eclipse:

$$M_E - M_0 = \frac{t_E - t_0}{P} \times 2\pi = v_E - v_0 - 2e\,(\sin v_0 - \sin v_E)$$

or, if v_E and v_0 are eliminated by the use of equation (1)

$$t_E = t_0 + \frac{(\omega_0 - \omega_E)}{2\pi} P + E \cdot P - \frac{eP}{\pi} \cos \omega_0 + \frac{eP}{\pi} \cos \omega_E.$$

All terms on the right-hand side of the last equation are constant except those that are linearly increasing with P, and the term in $\cos \omega_E$ which is variable if the line of apsides of the system is advancing. It would appear from the formula for t_E that it contains a sinusoidally varying term, and this indeed is usually assumed to be the case for regular advance of the apsidal line. The approximations made in deriving the formula should be noted, however. First, mid-eclipse only occurs at the true anomaly $\pi/2 - \omega$ if the inclination of the orbit is 90° and the stars are spherical. This latter condition is probably not very important, because even a distorted star presents a nearly circular cross-section to the observer at the time of mid-eclipse. The most important approximation is the neglect of terms in powers of e higher than the first in the series expansion of $v - M$. When e^2 can be justifiably ignored, e itself is quite small, and apsidal motion is difficult to detect. Thus the cosine term in the final expression is only an approximation. The true variation in the interval between times of minima is not perfectly sinusoidal, and may differ appreciably from a sine curve if the orbital eccentricity is large. The variable E is not a continuous variable, since it can assume only positive integral values. Kopal and Kurth (1957) have shown, however, that it may legitimately be treated as a continuous variable in the equations, provided that the period of apsidal motion, U, is very much greater than the orbital period. The times of secondary eclipse vary with the same amplitude as the times of primary eclipse, but in the opposite sense since the condition for secondary mid-eclipse is that $v = 3\pi/2 - \omega$, and the variable term

in the expression for the times of minima has the opposite sign (Fig. 4.4).

The amplitude of the variation in the times of minima is proportional to the orbital eccentricity e, since the maximum possible displace-

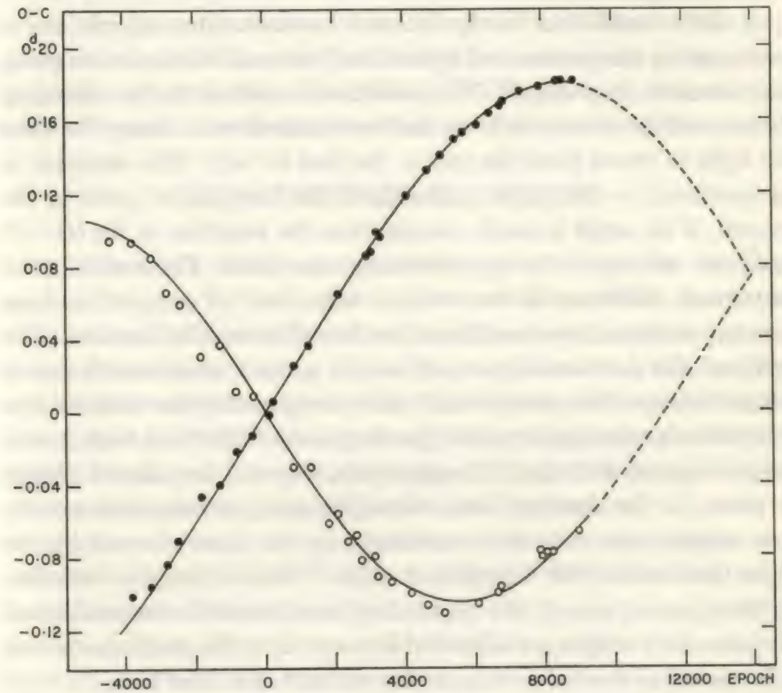

FIG. 4.4. Residuals of times of minima of V526 Sagittarii as determined and plotted by O'Connell (1971). Closed circles represent times of primary minima, and open circles secondary minima. The orbit has an appreciable eccentricity ($e = 0.244$).

ment of a minimum is eP/π. Observations of minima can be used, therefore, to determine a small orbital eccentricity. Since individual times of minima can be determined with an accuracy of 10^{-3} to 10^{-4} of a day, and since P/π is of the order unity for most eclipsing systems, it might be expected that apsidal motion could be detected, and a value of the eccentricity determined for e of the order of 10^{-3}. Few

observers, however, would claim to be able to determine eccentricities less than 0.01. In many systems, other causes of variation in the interval between minima are present, and can confuse the variation that would be produced by a very small eccentricity of an orbit in which the line of apses rotates.

A similar variation in the interval between times of minima is produced by the presence of a third body around which the eclipsing pair revolves (e.g. Algol). The variation is caused by the changing distance of the close pair from the Sun which in turn causes the time for light to travel from the star to the Sun to vary. This variation is proportional to the major semi-axis of the long-period orbit in the system. If the orbit is nearly circular then the variation in the $(O-C)$ residuals will again be approximately sinusoidal. There will be an important difference in the relative behaviour of primary and secondary minima, however, from that found for apsidal motion. The residuals for the two eclipses will not, in general, vary exactly out of phase because of the presence of a third body, as they do when the line of apsides is rotating. Provided that the period of the third body is very long compared with that of the close pair, they will vary almost exactly in phase, for the observed times of neighbouring primary and secondary eclipses are delayed or advanced by the same amount by the light time across the long-period orbit. When a periodic variation in the apparent period of a system has been detected, observations of the secondary eclipse are of crucial importance to the decision whether it is caused by apsidal motion or the presence of a third body.

All observed periods have a built-in light-time effect, since virtually all systems have radial velocities relative to the Sun that differ from zero. Kopal (1950) has pointed out that a correction of

$$P_{app} = P_{true}(1+V_0/c)$$

should be made to the observed period, where c is the velocity of light. The correction is small, but not negligible, since V_0/c is usually of the order of 10^{-4}. The period is used for predicting *observed* times of minima however, and there is little point in making a correction to the period that would have to be removed again before any comparison

with observation. Even for statistical studies of the distribution of periods, it would seem that this correction is unimportant.

Although apsidal motion and light-time in a long-period orbit are the two major causes of continuous periodic changes in the apparent orbital period, Kopal (1959d) has drawn attention to several other causes of apparent period variation. They also arise from a changing geometric aspect of the orbit; for example, one such cause is regression of the line of nodes. Plavec has shown (1960a), however, that most of these will produce unobservable effects on the times of minima.

REAL CHANGES OF PERIOD

Apsidal motion and the presence of a third body are not the only causes of variations in the intervals between minima. If the changes in these intervals are not themselves periodic, they are probably the result of real changes in the orbital period, rather than of a varying aspect of the orbit. Two kinds of real changes in orbital periods have already been mentioned: the apparently abrupt single change exhibited by RW Tauri; and a continuous change in the period of a system, of which the best-known example is provided by β Lyrae.

Evidence of abrupt changes in the periods of many systems was collected and published by Dugan and Wright (1949). Both increases and decreases of the period are known, and in many systems (e.g. U Coronae Borealis) both are observed. Early workers often tried to force all observed times of minima to fit power series or trigonometrical formulae, but it is now generally agreed that the periods of many systems do suffer genuinely irregular changes. The paper by Dugan and Wright was an important step towards this conclusion, because they found that it is often very difficult to fit the observed times of minima by analytic or trigonometric functions of the number of cycles elapsed. The magnitude of these changes is usually about one second; that is, about one part in 10^5 of the orbital period. The analysis of an earlier section, therefore, indicates that the effects of these changes should be readily detectable. This type of period change is characteristic of semi-detached (Algol-type) and W Ursae Majoris systems.

Plavec *et al.* (1960) were unable to find any evidence of such changes in well-observed detached systems.

Detre and Detre (1965) have applied the theory of random walk to the study of the periods of variable stars, including eclipsing variables. Small random variations in the intervals between successive minima can build up to a large, apparently systematic variation. Thus, some observed period changes may not be real, and, in particular, apparently periodic changes of period should not be considered as established until they have been observed over several cycles. Despite this need for caution in individual cases, the statistical evidence for real period changes is very strong, and there can be little doubt that they actually happen.

These period changes appear to be abrupt, although, as already mentioned, they may be very rapid continuous changes that act only for a short time. Occasionally they can convincingly mimic a periodic change of period—as, for example, in the so-called great inequality in the times of minima of Algol. There are at least three apparently periodic terms in the expression for the times of minima of this system. First, there is the term with a period of 1.8 years. This is well established as light-time in the long-period orbit. The second term has a period of about 33 years, and is almost certainly caused by apsidal motion of the orbit of the close pair. Finally, there is the great inequality, which at its maximum delayed eclipses by about three years. For a long time it appeared to be periodic, with a period of about 175 years. Scarcely that amount of time has elapsed since the discovery of Algol as an eclipsing binary, however, and the recent behaviour of the system has suggested that the great inequality is not periodic. Herczeg (1969) has suggested that it is the result of two abrupt changes of period of several seconds each. Apart from these, and the two well-recognized periodic terms, nothing else is needed to explain the observed variation in the times of minima. Attempts to explain the great inequality in terms of light-time in a long-period orbit about a *fourth* member of the system have always seemed rather unconvincing because the fourth body either should be easily visible, or must have very unusual characteristics.

Another interesting example is the system W Ursae Majoris. The $(O-C)$-time diagram for this system seems to show a discontinuity (Fig. 4.5). The period appears to be the same on either side of the discontinuity, but eclipses suddenly began to occur later than the expected time, by a constant amount (Cester, 1969). It is as if the phase of primary minimum has changed while the period remained constant.

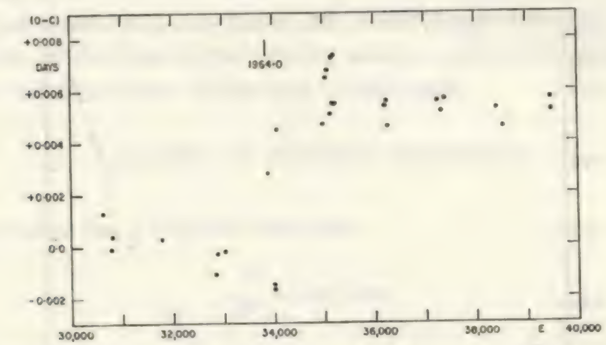

FIG. 4.5. Behaviour of times of minima of W Ursae Majoris showing the apparent change of phase of primary minimum around 1964. Based on data published by Cester (1969).

Perhaps there was a sudden change of period that soon afterwards reversed itself. The period of the system is less than a day, and even a short-lived change could easily be detected.

Dugan and Wright were hesitant to admit the reality of steady, continuous changes of period of the type that seems to be established for β Lyrae. They remarked that many systems that seemed to exhibit such changes were often found to suffer abrupt changes if observations were continued for long enough. An apparently steady change that has been observed for only a few decades may well have some surprises in store for the unwary theoretician. The period of β Lyrae, however, was first determined by John Goodricke (1785), and has been studied for a longer time than the period of any other system, except for that of Algol. A steady increase of about nineteen seconds a year (or six parts in 10^7 per cycle) seems to be well established (Rossiter, 1933).

Another example is provided by U Cephei, the period of which has now been studied for nearly a century. This period also seems to be increasing steadily, but at a much slower rate (Lukatskaya and Litovchenko, 1965). Eclipses observed in 1969 were about 15 hours later than expected from Wendell's ephemeris (1909) based on observations made around 1900. The present period (about 2.5 days) is approxi-

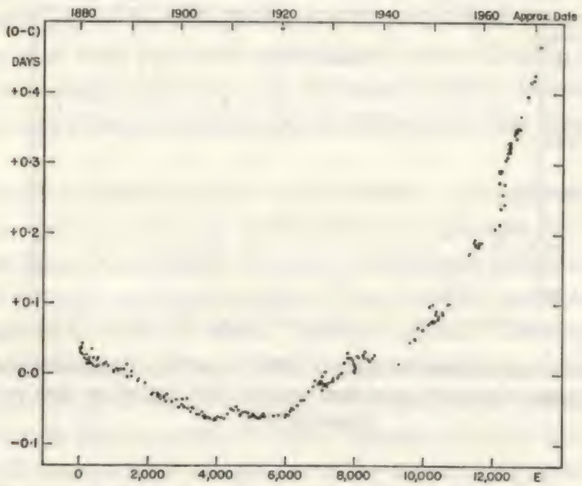

FIG. 4.6. Residuals of times of minima of U Cephei since its discovery as a variable star. Some individual minima have been omitted in crowded portions of the plot. The adopted period is $2^d.4929005$. The observations indicate that the present period is $2^d.49302$. The period has increased continually (but not steadily) throughout 90 years of observation. Most times of minima before $E = 11{,}000$ were collected by Svechnikov (1955). The remainder have been obtained from many sources, some unpublished.

mately 12 seconds longer than that found by Wendell. Intermediate observations indicate that this increase has been gradual and continuous (Fig. 4.6) and corresponds to an increase during each cycle, $\Delta P/P = 4.3\times10^{-9}$, although irregular fluctuations are probably superposed on this increase. As explained in an earlier section, a continuous change in period even smaller than this soon produces easily detectable effects on the times of minima, and there can be little doubt

that the period of U Cephei has increased over the past 80 or 90 years, whatever may happen in the future.

The individual times of minima of systems that show real changes of period often exhibit a rather large scatter in the $(O-C)$–time diagram. This can hardly be the result of observational error, because many of these systems display deep eclipses that should have been well timed. The light curves of many of these systems are distorted, and Kwee (1958) and van Woerden (1957) have both suggested that variable distortion of the light curve, during eclipse, could introduce errors into the determination of the time of minimum.

CAUSES OF PERIOD CHANGES

It is evident from Kepler's third law

$$\frac{a^3}{P^2} \propto m_1+m_2$$

that a real change in the period must imply a change of either the major axis of the system or the total mass, or both. If the major axis is assumed to remain constant, then

$$\frac{\Delta m}{m} = \frac{-2\Delta P}{P}.$$

The assumption can only be valid, however, if the *system* loses mass isotropically. Any other mode of mass loss from the system necessarily exerts forces on the component stars that will change the major axis as well as the period. If one component loses mass isotropically, it does not necessarily follow that loss of mass from the system will also be isotropic: the presence of the companion star makes the escape of matter harder in some directions than in others, and may even prevent escape from the system altogether. Isotropic loss of mass from a system as a whole, therefore, will rarely be observed. Two situations may approximate to it. One is isotropic loss of mass from one component of a very wide binary, such as α Herculis (Deutsch, 1956). The orbital periods of systems like this are too long for changes to be

observable. The other situation is isotropic loss from one component when the escaping matter has a very high speed. Binary systems containing Wolf–Rayet stars may furnish an example of this situation. There is spectroscopic evidence that mass is escaping from the Wolf–Rayet components at the rate of about $10^{-6}\ m\odot$/year, and velocities of the order of 1000 km/sec (Underhill, 1966). Little is known about period changes in such systems, but a recent investigation of two of them by Semeniuk (1968) does not yield any evidence of the expected steady increase of period. The companion stars are massive, however, and may deflect even high-speed matter. The overwhelming majority of period changes, if not all of them, must be explained by a more complicated mechanism than loss of mass from the whole system.

If one component of a system loses mass in a preferred direction, then the period can either increase or decrease, depending on that direction. If the loss of mass is supposed to take place at fairly high speeds (several hundreds of kilometres per second) then it can be quite efficient in changing the period of the system. The period of a system that contains a star similar to the Sun could be changed by about a second if the solar-type component were to lose about $10^{-7}\ m\odot$ in a process like that involved in an eruptive prominence. This hypothesis was first advanced by Wood (1950) to explain abrupt period changes. It was, at first, strongly attacked because the amount of ejected matter required seemed to be very large. Later, however, Huang (1956) and Piotrowski (1964) showed that transfer of mass between the components could be more efficient in changing the period than could simple loss of mass from the system, because the impact of the ejected matter on the receiving star also has an effect on the period. Moreover, it is now considered likely that gravitational forces are not the only, or even the dominant, interaction between the star and the ejected matter. Detre (1969) has suggested that ionized particles in the ejected matter can interact with the magnetic fields of the stars, and that this would be a very efficient way of changing the period. Meanwhile, advances in the theory of the evolution of binary systems, discussed in Chapter 10, indicate that quantities of matter of the order of 10^{-6} or $10^{-7} m\odot$/year are transferred from one component to another, and Wood's figures now seem more nearly credible. Thus, his hypothesis, modified to include non-gravitational interactions, is the most plausible explanation of abrupt period changes.

Surprisingly, perhaps, continuous period changes of the type found for β Lyrae are probably also results of mass transfer. Early investigators of β Lyrae attempted to relate the period change with loss of mass from the system, but the rate of increase of the period is too large to be accounted for by the amount of matter that appears to be escaping. The observed period change can be more easily explained by the hypothesis that a steady stream of matter flows from the less massive to the more massive component. If the total mass and angular momentum of the system are conserved in the process of transfer (i.e. if no mass escapes at all) it can be shown that

$$\frac{\Delta P}{P} = 3\,\frac{2\mu-1}{\mu(1-\mu)}\,\frac{\Delta m}{m} \tag{1}$$

where $m = m_1+m_2$, $\mu = m_2/m$, and ΔP is the change in period produced by the transfer of the amount of mass Δm. The suffix "2" denotes the star gaining mass, and Δm is always assumed positive. Thus the period increases when the flow of mass is from the less to the more massive component, and decreases otherwise. Sometimes it is useful to use the integrated form of the formula (Plavec, 1968a) which relates the initial (suffix "0") and final periods and masses

$$P/P_0 = (m_{1,0}m_{2,0}/m_1m_2)^3.$$

Differencing this formula gives equation (1), and shows that the right sign convention has been adopted. Both formulae are equivalent to a result first obtained by Kuiper (1941). The observed value of $\Delta P/P$ for β Lyrae is about 6×10^{-7}. If μ is somewhat arbitrarily assumed to be 0.667 (it must lie between 0.5 and 1) then

$$6\times10^{-7} = 3\times(3/2)(\Delta m/m)$$

or

$$\Delta m/m = 1.33\times10^{-7}.$$

The observed mass function is 8.5. If $\mu = 0.667$, and the invisible component is the more massive one (as is now generally supposed), then m must be about $30m_\odot$. Thus the observed period change requires the transfer of about $5\times10^{-6}m_\odot$ per orbital cycle, or about 11.4×10^{-5} $m_\odot$ per year. A similar calculation for U Cephei, in which the total mass of the system is assumed to be $10m_\odot$ and μ is again taken as 0.667, yields a transfer rate of $9.6\times10^{-9}m_\odot$ per cycle, or 1.4×10^{-6} $m_\odot$/year. In these systems it is necessary to postulate a steady stream of matter between the components, in order to explain the observations, while in others the process appears to be discontinuous. Both kinds of process may occur in some systems, as appears to be the case for U Cephei, and in those systems for which Dugan and Wright found that continuous changes in the period are interrupted by abrupt changes.

Both mass loss and mass transfer can either increase or decrease the period of a system. The effect of mass transfer depends on the direction of flow within the system: that of mass loss depends on the direction and speed of the escaping mass. A good review of the theory of period changes caused by transfer and loss of mass has been published by Kruszewski (1966). The study of period changes alone is not sufficient to show which of the two possible causes is operating in a given system. Mass transfer is discussed more fully in Chapter 9, after observational evidence for the existence of matter between the stars in a binary system has been presented and discussed.

If the components of a binary system move through a resisting medium, the orbital elements, including the period, will be changed. Intuitively, it seems likely that a system in which mass is being transferred, or from which mass is being lost, will be surrounded by a more or less tenuous cloud. Motion in a resisting medium tends to reduce the orbital period, although the reduction cannot easily be estimated quantitatively because the nature of the interaction between the star and the medium is not fully understood. In addition, the resisting medium may share the orbital motion of the stars from which it originated. Sobouti (1970) suggested that if there is matter surrounding the components of a binary system, currents must be set up in it,

and some form of frictional resistance between it and the stars appears to be inevitable. Some attempts were made to estimate this sort of frictional drag at the time that accretion of mass from interstellar clouds was thought to be important in stellar evolution (see, for example, Dodd and McCrea, 1952). Huang (1956) applied these results to the motion of a binary star in a resisting medium, and derived the formula

$$\frac{\Delta P}{P} = -(5+3\beta)\frac{\Delta m}{m}, \qquad (2)$$

where β is a parameter describing the frictional resistance, which Huang estimated to be about 10, Δm is the mass accreted by both stars, from the cloud, in each revolution, and m is the total mass of the system. The value of Δm depends on the distance that each star moves through the cloud, the area of stellar surface collecting matter, and the density of the cloud. Thus,

$$\Delta m \propto aR_*^2 np,$$

where a is the major semi-axis of the relative orbit, R_* is the sum of the radii of both stars, n is the particle density in the cloud, and p is the mean mass of the particles. If the orbit is circular, and the stars are spherical, the constant of proportionality is $8\pi^2$. Two stars, each of solar mass and radius ($R_* = 1.5\times10^{11}$ cm), moving in a circular orbit with $a = 7.5\times10^{11}$ cm, through a cloud containing 10^{11} particles/cm^3 will accrete approximately $10^{-10}m_\odot$ per orbital revolution. If $\beta = 10$, the orbital period should decrease in each cycle by one part in 10^8 or 10^9. This would be a continuous decrease, and should, therefore, be readily detectable. There are not so many well-known examples of continuous decreases of period as of continuous increases, which suggests either that clouds around systems are less dense than 10^{11} particles/cm^3, or any decrease in period caused by a cloud is more than compensated in most systems by mass exchange, or finally, that the mathematical treatment of the resistance between star and cloud is incorrect.

CHAPTER 5

STELLAR MASSES AND RADII

DETERMINATION OF STELLAR MASSES

One of the most obvious and fundamental contributions of double-star astronomy to astrophysics is the determination of the absolute dimensions of stars (masses, radii, and luminosities). Of the three quantities, the most important in the present context is the mass, because the only direct methods of mass determination are those connected with binary stars. The radii and luminosities of stars can be determined in other ways, although the contributions to our knowledge of these quantities that have resulted from the study of binary stars are certainly important.

The fundamental equation for the determination of stellar masses is Kepler's third law

$$\frac{a^3}{P^2} = m_1 + m_2,$$

where a is in astronomical units, P in years, and m_1 and m_2 in solar masses. It is necessary, therefore, to know both the period of the system and the major semi-axis in order to determine the total mass of a binary system. It has been shown in the previous chapter that the period can usually be determined with considerable accuracy (except, possibly, for long-period visual binaries) and the accuracy of the determination of the masses depends primarily on the accuracy with which the major semi-axis, a, can be determined. Unfortunately, this quantity appears cubed in Kepler's law and the total mass is therefore fairly sensitive to it. As a rough working rule, there is, proportionally, about 3 times the uncertainty in the total mass that there is in the semi-axis. Thus accurate determination of the total mass of a system is very difficult. Many investigators are content to use a large

number of data of inferior accuracy, hoping that the uncertainties will be random and that useful conclusions can be drawn. No doubt many problems can be tackled in this way, but others (e.g. tests of theories of stellar evolution, discrimination of possible differences in the mass-luminosity relations of different clusters) require accurate values of masses for their solution, and particularly masses that are free of systematic error.

For a visual binary, the orbital element a'' can be determined directly, and this can be converted into an absolute value of a if the parallax π'' is known. For a spectroscopic binary with both spectra visible, the element $a \sin i$ can be determined from the observed amplitude of velocity variation K_1+K_2. This can be freed of the $\sin i$ term if the binary is an eclipsing system. The parallax of a visual binary can be determined either directly, as it is for single stars, or from radial-velocity observations, since the relative radial velocity of the two component stars is given by

$$V_r = \frac{29.76\, a'' \sin i}{P\pi''(1-e^2)}\; e \;\cos\omega + \cos(v+\omega) \quad \text{km/sec.}$$

Thus, in principle, one determination of the relative radial velocity of the two components of a binary system, for which the orbital elements are known, serves to determine the parallax and the total mass of the system. In practice, of course, observations should be made all round the orbit.

To obtain the individual masses, another function of them must be determined. This is usually the mass ratio m_2/m_1. The mass ratio of a visual binary must be obtained either from astrometric measures of the absolute orbits of the two components, or again from radial velocities. If the radial velocities of the two components are determined, then the system is also a spectroscopic binary for which the mass ratio is given by

$$m_2/m_1 = K_1/K_2,$$

and since K_1 and K_2 are orbital elements of a spectroscopic binary, m_1 and m_2 can quickly be determined separately.

Visual binaries that show sufficient orbital motion for the period and orbital elements to be accurately known are, for the most part, relatively nearby systems (see Chapter 2). Most of them, therefore, contain stars of low mass, because such stars are the most common in the solar neighbourhood. Most of our knowledge of the masses of stars less than one solar mass is derived from visual binaries. Eggen has pointed out (1962, 1967a) that the errors in a^3/P^2 may be quite small even though a and P themselves are only poorly known. Thus it might be possible to determine masses from visual binaries that have been observed only over a portion of their orbits, especially if the observed arc contains one of the nodes. Among such systems there may well be included some more massive stars, but prudence suggests that critical determinations of mass should only be made from those systems in which the period is well known.

Radial velocities have been determined for the components of only relatively few visual binaries. The reasons for this are given in Chapter 3 in the section on spectroscopic-visual triples. Improvements in spectrograph design are making it increasingly possible to obtain high dispersion spectrograms of fainter stars. At a dispersion of 2.5 Å/mm, velocity differences of as little as 10 km/sec can be measured reasonably accurately for pairs of stars of late spectral type. Spectrograms of this dispersion can now be obtained for stars as faint as seventh magnitude.

Because of selection effects discussed in Chapter 2, double-lined spectroscopic binaries tend to be found amongst the hotter, more massive stars. Therefore, our knowledge of masses greater than a solar mass is largely derived from spectroscopic eclipsing systems, and that of the largest stellar masses is exclusively derived from such systems. Indeed, few stars with known masses greater than $2m_\odot$ are found in visual binaries. The determination of masses from eclipsing systems is also subject to the uncertainty in sin i. If a system shows eclipses at all, however, its orbital inclination must be fairly high, and sin i is then not very sensitive to the precise value of i itself. Thus the chief source of uncertainty in the masses is still that in K_1 and K_2.

SYSTEMATIC ERRORS IN MASS DETERMINATION

This section is concerned with various systematic errors that can affect the determination of masses from *spectroscopic* (eclipsing) binaries. Observations of visual binaries are, of course, subject to systematic errors that differ for different observers (Aitken, 1935b). They can be usefully discussed only by one who has himself had experience of measuring visual binaries. Most visual-binary orbits must be determined from the observations of many different people, because of the length of the orbital period. Thus the systematic errors of different observers, if they are not allowed for, become random errors in the final orbital solution, and their effect on the orbital elements is minimized. On the other hand, spectroscopic orbits are usually determined by one observer at one observatory, using one instrument. It is even considered desirable to achieve such a degree of homogeneity in the observations. A careful discussion of systematic errors is, therefore, all the more important.

EXPOSURE-TIME EFFECT

If the exposure time for an average spectrum is an appreciable fraction of the orbital period, then the radial velocity of each component will change appreciably during the exposure, and an average velocity will be measured. As a result, the total range of the velocity variation will be underestimated. Schlesinger (1916) discussed this quantitatively. The effect is usually unimportant except for systems with periods less than 1 day (e.g. the W Ursae Majoris systems). In such systems, K_1 and K_2 may be frequently underestimated by 5 to 10 per cent, and occasionally much larger errors may arise.

PAIR BLENDING

The distortion of the profiles of two components of a spectral line that are not properly resolved is mentioned in Chapter 3. The minima of the combined line profiles are closer together than are those of the

two separate profiles (Fig. 5.1). If a measurer tries to set on the minima of the combined profile, he will underestimate the true separation of the two components of the line, and, therefore, the velocity difference between the two stars. This blending of the same lines in different spectra can conveniently be called *pair blending* to distinguish it from

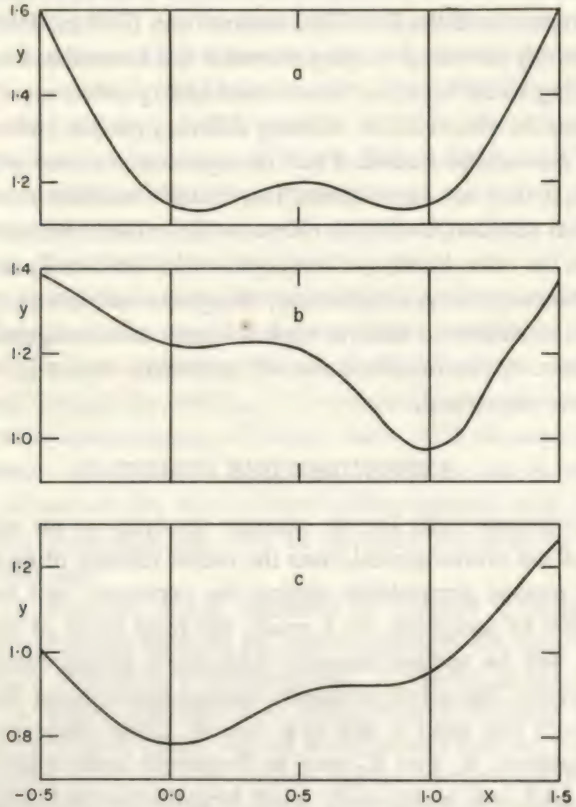

FIG. 5.1. The effect of pair blending on the measured separation of two lines, as illustrated by Tatum (1968). The vertical lines indicate the true separation.

the ordinary blending of different lines in the same spectrum. The errors in velocity introduced by this effect lead to errors in the values derived for the masses of a binary system.

The simplest way to investigate pair blending is to make measurements of artificially doubled spectra for which the true separations are known. This has been done by Petrie *et al.* (1967) and Batten and Fletcher (1971). The earlier of these investigations was concerned with the measurement of spectra of late B and early A types. In these spectra, the hydrogen lines have their maximum strength, and are often the only features that can be measured. The line profiles of binary spectra of this type, therefore, may not be fully resolved at any phase. The other investigation was concerned with late-type stars whose spectra contain many, sharp lines. The components of many visual binaries have spectra of this type, and their velocity differences may be so small, that even these sharp line-profiles cannot be properly resolved. Both investigations have led to very similar results because, as was to be expected, the effects of pair blending depend on the ratio of the width of the individual lines to the separation of the pair, and not on the absolute width of the line. No appreciable systematic error is made in the measurement of a line pair separated by more than 1.5 to 2 times the half-widths of the component lines (by "half-width" is meant the total width of the line at half its depth). This observational result is confirmed by computations made by Tatum (1968). All separations less than this are measured too small, the error increasing as the true separation decreases. When the true distance between the minima of the line profiles is equal to, or less than, the half-width, the error in measurement may reach 20 or 30 per cent. Thus considerable errors in the estimated total mass of a binary system may arise from pair blending.

Measures made with an oscilloscopic setting device appear to suffer a larger systematic error as a result of pair blending than do those made by a trained visual observer. This is probably because the measurer looking at an oscilloscope display sets on the actual minima of the combined line profile. The visual measurer cannot see the wings of the lines, and often believes that he is measuring two well-resolved components. He sets on the centres of these, and by doing so, because their profiles are asymmetric, he partially compensates for pair blending. Different people make visual measurements differently, however,

while measures made with an oscilloscope are more impersonal. Thus the oscilloscope results, although wrong, are predictably wrong, and it is probably safer to measure close line pairs with an oscilloscope device whenever possible, and to apply a computed correction to the measurements. Although oscilloscope comparators enable one to set on a given line more certainly, and with greater precision, they do not necessarily yield radial velocities of any greater accuracy than do other means of measurement. The large "measuring errors" found for stars of early spectral types are not strictly measuring errors, but, at least partly, intrinsic to the spectrogram.

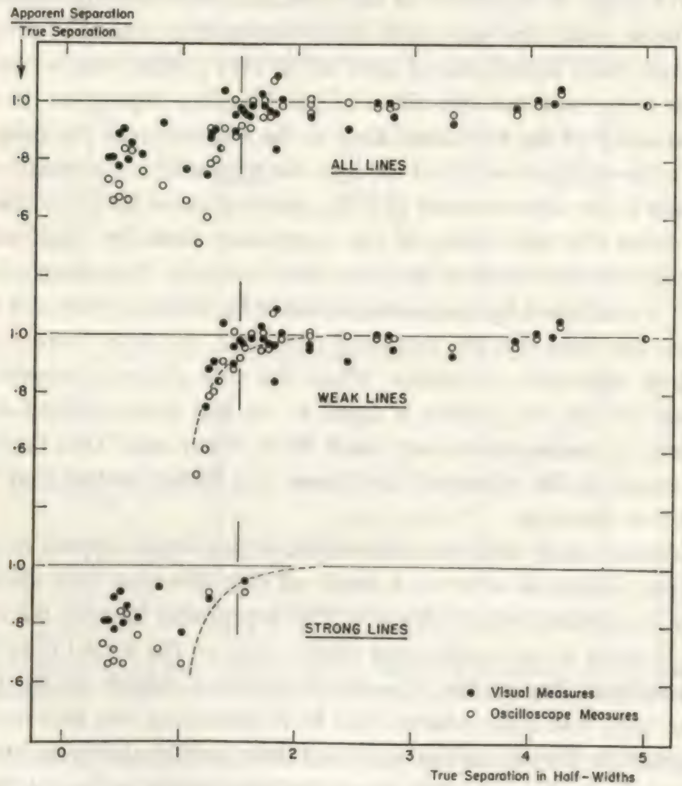

FIG. 5.2. Measured effects of pair blending on the positions of artificial double lines of known separation.

Strong and weak line pairs in artificially doubled solar-type spectra are affected differently by pair blending. As long as they can be measured at all, the stronger pairs are less affected at a given relative separation. This difference—illustrated in Fig. 5.2—is probably a result of the different-shaped line profiles. Those of the weak lines are dominated by the instrumental profile of the spectrograph, while those of the stronger lines are more nearly the natural profiles.

Pair blending in spectra of early A or late B spectral types is a consequence of the great intrinsic widths of the only measurable lines, and cannot be entirely avoided by the use of spectrographs of higher dispersion (see Plate I (b)). Spectra of later types usually contain lines of much smaller intrinsic width, and the actual measured width may be determined by that of the spectrograph slit. Pair blending in these spectra may be removed at all but the smallest separations, if a spectrograph whose resolution is limited only by that of the photographic emulsion is used. One would therefore expect that larger values will be obtained for K_1+K_2 (and the total mass) in a given system when higher spectrographic dispersions are used, at least if the component spectra are of late types. Popper (1965, 1967), however, has drawn attention to a number of instances in which the reverse is true. Popper's explanation appears to be pair blending, but this has now been shown to operate in the wrong sense. Batten and Fletcher (1971) found some evidence that the measured separation of a pair of lines depends partly on the image size of the spectrogram being measured, and this effect, if it exists, operates in the right way to produce Popper's result. It has not been fully studied, however. The effects of pair blending can be minimized if spectrograms are obtained only at the nodes of the velocity curve, when the true line separations are greatest. This has been Popper's practice, but it is also important to observe the whole velocity curve in order to be sure that it is not distorted. Popper has often stressed the importance of obtaining spectrograms of the highest possible dispersion when masses are to be determined, and this appears to be the best way to eliminate systematic errors that may act in either direction.

CONFUSION OF COMPONENT SPECTRA

Another reason for using high dispersion, emphasized by Popper, is that lines in the spectrum of one component may blend, at low dispersions, with different, oppositely displaced lines in the spectrum of the other. This kind of blending is more important in systems containing components with late-type spectra that contain many lines, and at low dispersions, when these lines are crowded together. In extreme cases the secondary spectrum may be invisible, and faint lines of the primary spectrum be mistaken for it. Although it seems unlikely that such an error should be committed on a whole series of spectrograms without producing nonsensical results, Popper (1967a) has shown that the anomalously low masses previously derived for the components of TU Monocerotis, can be explained as a result of this sort of confusion. The various lines in the combined spectrum of a system are not necessarily affected in the same way, and if many pairs of double lines are measured, the mean separations deduced for all the line pairs on the plate may be nearly correct; but it will be subject to large uncertainty if care has not been taken to eliminate this kind of blend from the measurements. The effect is well illustrated in Table 8, which gives the results of the measurement of three plates of the double-lined binary H.D. 27149 (G5) obtained at approximately the same orbital phase, but at different dispersions. The standard deviations of individual line measures should be roughly proportional to the dispersion of the spectrograph used if accidental errors of measurement only were operating.

TABLE 8. MEASURES OF VELOCITY DIFFERENCE OF THE COMPONENTS OF H.D. 27149

Dispersion (Å/mm)	No. of lines	Velocity difference (km/sec)	Standard deviation of single line	
			Primary (km/sec)	Secondary (km/sec)
15	12	74.7	4.8	11.6
10	9	75.8	2.8	7.8
6.5	5	77.8	1.7	1.8

DISTORTION OF THE VELOCITY CURVE ("REFLECTION EFFECT")

If a velocity curve is distorted, then the masses derived from it will be in error because they depend both on K (the amplitude of the velocity variation) and e (which specifies the shape of the velocity curve). As mentioned in Chapter 1, the orbital elements of many spectroscopic binaries are apparently variable, especially the element K. Because the observed changes often cannot be accounted for by the dynamics of the system, they are evidence that at some epochs the velocity curves of these systems are distorted. Some of these distortions are probably caused by photographic effects of the kind just discussed. Others arise because there are other things in the system, besides the two component stars; for example, a third body (as discussed in Chapters 1 and 3), or a gas stream between the components (mentioned in Chapter 1, and discussed in more detail in Chapter 8). Another possible cause of distortion in the velocity curves is an asymmetric distribution of luminous intensity over the surface of one or both of the stars in the system. The effective light centre of the apparent disk of the star may then be displaced from the geometrical centre to a point having a different velocity, and in addition, the profiles of the lines in the star's spectrum may also be asymmetrical, and, for both these reasons, the measured velocity of the star will be in error. This subsection is concerned with distortions of the velocity curve arising from this source.

Errors may be made in the measurement of the velocity of a distorted star because of the variations of temperature and surface luminosity over the surface of such an object. The most important source of non-uniform distribution of luminous intensity over the surface of a binary component, however, is the so-called "reflection effect". As explained in Chapter 1, light incident on one star from the other is re-radiated and contributed to the total variation of the light of the

system. The hemisphere of the reflecting star that faces its companion is heated by the incident light, and the temperature and luminosity distribution over the apparent disk of the star is no longer symmetrical. The effective light centre of the disk is displaced inwards, and its velocity is less than the orbital velocity of the centre of mass of the star—particularly if the star rotates on its axis more rapidly than it revolves in its orbit. This displacement of the light centre, and its effect on the estimated mass of the system, has been theoretically derived for bolometric radiation (Batten, 1957). It is most important for the fainter star of a pair in which the components differ appreciably in brightness. It cannot be computed by analytical methods, even approximately, for very distorted stars. Ovenden (1963) has pointed out, however, that radial velocities are not measured in bolometric light, but in the light of particular spectral lines. The strengths of these lines depend on the temperature of the stellar atmosphere in which they are formed, and some lines are very sensitive to temperature. Thus some lines in the spectrum of the secondary component may originate only in a small "cap" of the secondary star, immediately opposite the primary (see Fig. 5.3), while others may originate predominantly in the cooler hemisphere. Velocities measured from the first group of lines may be even smaller than would be computed from the bolometric formulae, while those from the other group of lines may

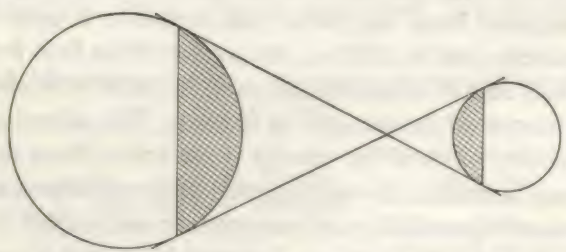

FIG. 5.3. A high-temperature cap can be formed on the facing hemispheres of each component of a close binary system. The shaded areas indicate those portions of each stellar surface on which the companion is fully above the horizon. If rotation of the components is synchronous with orbital revolution these sections of the surfaces may be heated considerably, and temperature-sensitive spectral lines may originate only in these sections.

actually be larger than the true orbital velocity. Ovenden found that values of K_1+K_2 derived from different lines in the spectrum of 57 Cygni (B3+B3) are widely different, and the maximum value derived for the total mass of this system is more than 3 times greater than the minimum. Lines normally formed only in atmospheres of higher temperatures than those that exhibit a B3 spectrum give smaller values for the total mass than do the lines that are also found in B3 spectra. This suggested that the dependence of K_1+K_2 on the lines measured in the spectrum of 57 Cygni is a result of "reflection", but Ovenden and Napier (1970) were unable to account for the observations quantitatively on this hypothesis, and suggested that turbulence might be produced by the local heating of a part of the "reflecting" atmosphere and should also be taken into account.

The effects of reflection and distortion can be computed by calculating the temperature and radiation at each point of an array over the surface of a star, and thus obtaining an approximation to the distribution of these quantities. If additional assumptions are made about the structure of the stellar atmosphere, then even the profile of a line produced in the "reflecting" atmosphere can be computed and the error in a velocity obtained from that line can be estimated. Computations of this sort are basic to the methods of computing synthetic light curves that are discussed in Chapter 7.

The errors introduced into the determination of total mass by "reflection" can be considerable, and usually lead to an underestimate. They are probably most important in systems containing at least one hot star: cooler stars like the Sun do not distort the temperature distribution over each other's surfaces so much.

RELIABLE MASSES

If reliable values of stellar masses are required, then they should be derived only from the best-observed systems. If the velocity curve shows any sign of distortion, or if the lines in the secondary spectrum are weak or uncertain, then the system is not suitable for accurate mass determination. Even if a system is satisfactory, care must be taken to

eliminate the systematic errors discussed in the preceding section. Unfortunately, there are many systems in which the velocity curve of the primary component is well determined, but that of the secondary is not. For example, the eclipsing system V 380 Cygni has been observed independently at three different epochs, and the primary velocity curve is probably one of the best determined for stars with early B-type spectra. The secondary spectrum, however, has been measured only on the spectrograms obtained at the latest epoch (Batten, 1962), and because it is much weaker than the primary spectrum the velocities determined from it show a much larger scatter. In addition, the light curve is poorly observed, and the orbital inclination is consequently not well known. Unless new observations of the secondary spectrum can increase the certainty of the velocity curve, the system is not suitable for a critical determination of the masses of the components. In some systems, such as Plaskett's star (H.D. 47129) and H.D. 190967, the "secondary" spectrum seems to arise at least partly from a non-stellar source (Fig. 8.5). No matter how well they may be observed, it will be impossible to obtain accurate masses for these systems.

In the choice and careful investigation of systems that are likely to yield reliable values of stellar masses, Popper has set standards of thoroughness and excellence that it would be difficult to surpass. The results of his critical selection have been presented by him elsewhere (1967b, 1970) and little is to be gained by reproducing his lists here. He refers to it as a "conservative list", and other investigators have published very much more extensive lists. Critical comments have been published on these lists by Popper himself, and he is undoubtedly right in insisting on the highest possible accuracy for this important basic datum. Perhaps α Virginis can be added to his list (Herbison-Evans *et al.*, 1971).

The results obtained by Popper from the study of eclipsing binaries have been supplemented by a number of well-determined values for the masses of visual binaries. Perhaps the most useful recent tabulation of these masses is that given by Harris *et al.* (1963) who give masses for the components of forty-one visual binaries. Their figures for Σ 2173 can now be modified (Batten *et al.*, 1971).

Good values have been obtained for masses of stars along most of the main sequence, although considerable uncertainty still remains at the upper end. The masses of giant stars are still uncertain, for the number of systems containing giants that fulfil the conditions for accurate mass determination is very small, and only one giant and two supergiants figure in Popper's list. As is to be expected, most of the binaries for which good masses can be obtained consist of two main-sequence stars, well separated, and without gaseous streams between them. There are no good direct determinations of the masses of Algol-type systems, and the determination of masses of contact systems also presents many problems.

LIMITS OF STELLAR MASSES

In 1959 Schwarzschild and Härm gave theoretical reasons for supposing that stars of more than about $65m_\odot$ would be unstable with respect to small vibrations created by nuclear energy generation, and that very massive stars would probably disrupt. This investigation stimulated Sahade to discuss the most massive stars then known (1962). He concluded that Plaskett's star, the most massive binary known, had components with masses of about $60m_\odot$, but there was no evidence for the existence of any more massive stars. Later, Peery (1966) estimated masses of $84.4m_\odot$ and $41.3m_\odot$ for the two components of VV Cephei. This was a very uncertain determination, because there appears to be some distortion of the velocity curve of the primary star, and the mass ratio of the system was determined from measures of only one emission line (Hβ) that can only tentatively be identified with the secondary spectrum. More recently, Wright and Larson (1969) have measured the emission line at Hα, and the velocities obtained from this indicate that the mass ratio is much closer to unity than the value Peery found. Even if the mean of the two possible values of the mass ratio is taken, the individual masses would be reduced below the limit of $65m_\odot$. On the theoretical front, Appenzeller (1970) has investigated the stability of a star of $130m_\odot$, and found that it loses mass only slowly. He believes that P Cygni may be an example of such a

massive star. Observers and theoreticians alike can no doubt take pleasure from the fact that no star has yet been found that ought not to exist.

A list of the most massive stars known was published by Batten (1968c). A number of new massive stars can be added to it now, in particular H.D. 35921 (O9) discovered to be an eclipsing binary by Mayer (1968) and known to show double lines in its spectrum. No systems have yet been discovered, however, that change the conclusion that Plaskett's star is probably the most massive system known, or that the most massive system for which masses can be derived without inference of the value of K_2 or the orbital inclination is V 382 Cygni, found by Pearce (1952) to have masses of $37m_\odot$ and $33m_\odot$. A recent determination of the mass of γ^2 Velorum by Ganesh and Bappu (1967) indicates that the O-type component of that system has a mass of $45m_\odot$. This is the largest completely determined mass of an individual star. The mass of the Wolf–Rayet component, however, is only $15m_\odot$, and the total mass of the system is thus inferior to that of V 382 Cygni. Neither of these systems qualifies for inclusion in Popper's critical lists of masses.

It is also interesting to consider the smallest known stellar masses, although this is to some extent a matter of definition. Kumar (1963) showed that the lowest value of the mass of a main-sequence star (i.e. one that can burn hydrogen in its interior) is between $0.07m_\odot$ and $0.1m_\odot$, depending on its chemical composition. Objects of lower mass can form the interstellar cloud. They become degenerate without going through a hydrogen-burning phase. The smallest mass that can condense at all depends on the density of the cloud. Kumar (1969) estimates it at $0.001m_\odot$. Salpeter (1969) defines the limiting mass of a star as the one at which Coulomb forces between elementary particles become important, that is about $0.003m_\odot$. Since the mass of Jupiter is only slightly less than $0.001m_\odot$, the choice of definition has one interesting effect—on Kumar's definition the Solar System can be regarded as a double star.

The gradual transition between stellar and planetary masses is more obvious when one considers the low-mass companions now known for nearby stars. Kumar (1969) lists eight, with masses ranging

from $0.002m_\odot$ to $0.05m_\odot$. The smallest is the unseen companion of Barnard's star, studied by van de Kamp (1963). It could be replaced by two objects, each of about the mass of Jupiter, revolving around the parent star. If there are two such objects, their orbits would be nearly circular, whereas if only one object is assumed to exist, its orbit must be quite eccentric. Possibly the dark companion found for Ci 2354 by Lippincott (1967), that also moves in a highly eccentric orbit, could be similarly replaced by two or more bodies moving in circular orbits. Since low-mass companions are very difficult to discover, it is highly probable that many more exist than are actually known. It has often been speculated, perhaps most recently by Huang (1966a), that planetary systems and binary systems are related objects. The present indication that there is no clear demarcation between planetary masses and stellar mass tends to support that speculation. Fracastoro (1969) has advanced the hypothesis that a complete continuity of mass distribution exists for all celestial bodies, from the most massive stars down to micrometeorites. He suggests that the number of objects of a given mass m is approximately proportional to $m^{-0.8}$.

THE MASS-LUMINOSITY RELATION

The mass-luminosity relation has probably come to seem less important during the last decade as we have become accustomed to the idea that a star of given mass can have very different luminosities in the course of its evolution, and the important relation is between the three parameters mass, luminosity, and radius. Nevertheless, the empirical mass-luminosity relation of main-sequence stars is still an important datum to which model calculations must conform, and its derivation depends almost solely upon binary stars. It is also important to establish the reality, or otherwise, of apparent differences between the mass-luminosity relations for different groups of stars. Comparison with theory requires the absolute bolometric magnitudes, whereas one normally determines visual magnitudes directly from observation (except for eclipsing binaries, for which the bolometric magnitude can be determined directly if the effective temperature is known). There is still

much uncertainty about bolometric corrections for stars of spectral types very different from the Sun, although this is now being reduced by data obtained from outside the atmosphere of the Earth. In particular, the bolometric corrections for the hot, luminous, and massive O and B stars are uncertain, so the upper part of the mass-luminosity relation is still poorly known. The luminosities of these stars, of course, are also uncertain, and the fact that these stars evolve more quickly (though fortunately at more nearly constant luminosity) than other stars makes it harder to determine a unique mass-luminosity relation.

A most thorough investigation of the relation between masses, luminosities, and radii was undertaken by Kuiper (1938 a, b). He found a change in slope for the relation between $3m_\odot$ and $10m_\odot$, although this is largely because he used Trumpler's massive cluster stars, for which the masses were derived on the assumption that their spectra showed a relativistic red shift, to define the upper end of the relation. This explanation of the apparent red shift in the spectra of O-type stars is no longer accepted, and the masses of Trumpler's stars are unknown. Kuiper also found a change of slope in the relation at about $0.6m_\odot$, and that the Hyades stars of about solar mass, or slightly less, are brighter than other stars of the same mass. He concluded that there is not a unique mass-luminosity relation, but rather several, and that the factor causing the difference is chemical composition of the stars, and in particular their hydrogen content.

Another important discussion, at about the same time, was that by Russell and Moore (1939). They found that no stars, other than the white dwarfs, deviated by an important amount from a simple linear relation between log and the luminosity, over the range of masses approximately $0.3m_\odot$ to $30m_\odot$. However, the quantity they use to measure luminosity is the bolometric magnitude plus

$$2 \log (T_e/5200),$$

as used by Eddington (1962a) in his theoretical work to reduce the data for all stars to the assumed effective temperature of Capella (T_e = 5200°K), and this should be remembered when their results are compared with Kuiper's.

Petrie (1950b) determined the slope of the mass-luminosity relation from all those double-lined spectroscopic binaries for which he had determined the magnitude difference, Δm, by his spectrophotometric method. Since Δm and the mass ratio are known for each system, a relation exists that can, in principle, be integrated to give the mass-luminosity relation itself, provided that the constant of integration can be determined from a few objects of well-known mass and luminosity. The advantage of this method is that the observations of many more binaries can be used since the determination of m_2/m_1 and Δm are independent of any knowledge of the orbital inclination. A further advantage is that integration of an empirical relation minimizes the effect of observational error. Its chief disadvantage is that spectrophotometrically determined values of Δm may sometimes be subject to large error. Petrie's relation, however, agreed well with previous determinations, and this suggests that no large systematic errors exist in his Δm material. He found that a single, nearly linear relation would satisfy all spectroscopic binaries in the absolute-magnitude range $+3^M$ to -5^M. (He also included the correction term $2 \log T_e/5200$ in the plot of the relation.)

Kopal (1959) also attempted to define an empirical mass-luminosity relation from the data of eclipsing and visual binaries. He found that two linear relations were needed, one for masses less than $2m_\odot$, and one for larger masses. The break occurs at roughly the mass for which the carbon–nitrogen cycle of energy generation becomes more important than the proton–proton cycle, and Kopal suggested that the break is a result of this change. According to Iben (1967) the theoretical mass-luminosity relation also shows a change of slope at this mass. Masses less than $2m_\odot$, however, are for the most part determined from visual binaries, while larger masses are more often determined from eclipsing binaries. Well-observed visual binaries should give masses largely free of systematic errors, and since their parallaxes are well determined when their total masses are known, they should give well determined points in the mass-luminosity diagram. Moreover, many of them have spectral types in the range within which the bolometric corrections are well known. On the other hand, the masses of

eclipsing binaries are subject to the systematic errors already discussed. The luminosities are not directly determined, but are found from the radius and effective temperature of the star. Although the bolometric correction does not enter explicitly into the determination of these luminosities, the scales of bolometric corrections and effective temperatures are, of course, closely related. Kuiper and Petrie both investigated the effect that errors in mass determination caused by the "reflection" effect would have on their mass-luminosity relations. Each of them found that this correction was important only for a very few systems. The correction for the blending of the wings of the hydrogen lines in early type spectra was not considered, and this would act in the right direction to remove the change of slope. Thus it is questionable whether or not the change of slope in the empirical relation near $2m_\odot$ has been fully established. In what is likely to be regarded as a "standard" version of the empirical relation for some time to come, Harris *et al.* (1963) confirm the change of slope near $0.6m_\odot$ that Kuiper found, but find no strong evidence of a change of slope near $2m_\odot$. In Fig. 5.4 a plot of the mass-luminosity relation is shown that is very similar to that given by Harris, Strand, and Worley. It is based on the same visual binaries as they used, except that some of the faintest components have been omitted, and new values used for Σ 2173. The spectroscopic binaries for which colours, masses, and radii are given by Popper (1970) have also been plotted. It has been necessary to assume the relation between intrinsic colour and effective temperature. In addition, points corresponding to Y Cygni, U Ophiuchi, YY Geminorum, and the Sun have been plotted. There is little evidence for a change of slope near $2m_\odot$, especially in view of the uncertainties in the bolometric magnitudes of the most luminous stars.

Recently, Eggen (1960, 1965) has rediscovered the apparently greater brightness of the visual binaries in the Hyades, as compared with the standard mass-luminosity relation, that was first pointed out by Kuiper. Like Kuiper, he ascribes this to a differing chemical composition, which he believes to be brought about by evolution. The lower main sequence appears to be about a quarter of a magnitude brighter, for the Hyades, than that of stars of the same mass range in

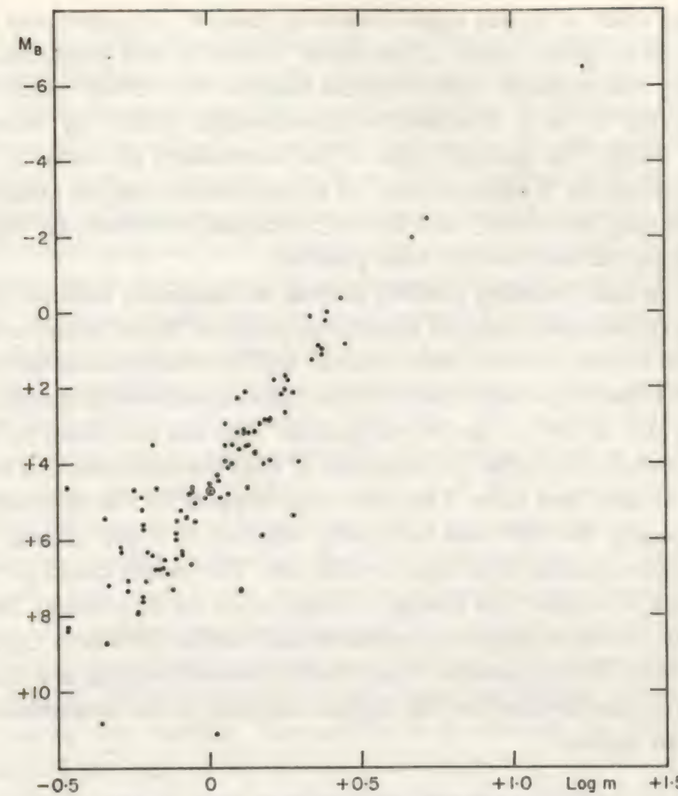

FIG. 5.4. The empirical mass-luminosity relation.

the so-called Sun–Sirius group. It is not clear whether Eggen believes the Hyades to be the older or the younger group, as he has published conflicting statements on this point (1960, 1965). Wallerstein and Hodge (1966) have challenged the existence of this dichotomy in the relation, but since their solution involves changing the accepted distance modulus of the Hyades, which in turn has ramifications for the extragalactic distance scale, it has not yet been generally accepted. Popper has emphasized the very small changes in parallax that would suffice to bring the two empirical relations into agreement (1967b). Heintz (1969b) has pointed out that the visual binaries in the Hyades

do not define a unique mass-luminosity relation. The discovery of a late-type eclipsing binary in the cluster would be very important for settling this problem. Such a system may be H.D. 27149 which was discovered to be a double-lined spectroscopic binary by Woolley *et al.* (1960). The spectral types of the components are early G, and observations by Wallerstein and the writer indicate that the minimum masses may be $0.96m_{\odot}$ and $0.84m_{\odot}$. Eclipses, therefore, are highly probable, but have not yet been detected.

Batten and Ovenden (1968b) derived an empirical relation from all the eclipsing binaries and visual binaries in the *Sixth Catalogue* that showed double lines in their spectra and for which parallaxes were known either trigonometrically or from the measurement of the equivalent width of Hγ in the B-type spectra. This was not meant to be a critical study, but rather an indication of what the data indicated when taken at their face value. They give some support for the existence of two roughly parallel mass-luminosity relations at lower masses, but only more accurate data can confirm this. The results could as well be taken to support the change of slope found by Kopal near $2m_{\odot}$, and this has more support from theoretical models than does Eggen's hypothesis. This hypothesis is important and controversial, and serves to emphasize the need for the highest accuracy in the determination of stellar masses.

DETERMINATION OF STELLAR RADII

Except for a few supergiants whose radii had been determined by Pease (1931) using a Michelson interferometer on the 100-inch telescope, the radii of single stars could not be determined at all until quite recently. The Michelson interferometer mounted on a large telescope proved to be an unwieldy instrument, and Hanbury Brown and Twiss (1956) modified the interferometer technique to use two large mirrors feeding photomultipliers whose signals are correlated electronically. An instrument of this kind has been used to measure the angular diameters and effective temperatures of fifteen stars (Hanbury Brown *et al.*, 1967) including main-sequence stars. Parallaxes are available for six of these stars, and linear diameters have been obtained for them. In principle, the technique can be applied to any star, and is therefore potentially more useful than the determination of stellar radii from eclipsing binaries. It makes considerable demands on observing time—a faint star may have to be observed for 50 hours to build up a sufficient signal for the determination of its radius. This is comparable, however, to the times needed to obtain a good light curve, and it is much to be hoped that a larger instrument can be built in order that many fainter stars can be observed. Although errors can be introduced into the measured diameters by the presence of faint unresolved companions, any such companion that is bright enough to affect the results should be easily detectable. The results obtained so far are in encouraging agreement with expectations based on eclipsing-binary data. Recently, Herbison-Evans *et al.* (1971) determined the radii of the components of α Virginis with this instrument.

Another possible method for the direct determination of the diameters of single stars is the observation of occultations of stars by the Moon. The method was described and attempted by several people; its history is reviewed by Evans and Nather (1970). Some time ago Evans *et al.* (1953) attempted to determine the angular diameter of Antares by this method. The method is to observe the diffraction pattern of the light of the star as it is occulted by the Moon. The characteristics of this pattern are dependent upon the angular diameter of the star. The method promises well for some stars for the future, although our knowledge of the detailed profile of the lunar limb is not yet sufficiently precise, and few results have been obtained.

The principal method of determining stellar radii directly remains, for the time being, the analysis of light curves of eclipsing binaries. As explained in Chapter 1, the radii of stars, in terms of their separation as unit, can be obtained from the light curve of an eclipsing binary system. If spectroscopic observations of the system have also been made, and two spectra are visible, then the separation of the two stars can also be determined, and their radii can be obtained in kilometres. The determination of radii depends also on the limb-darkening of the star being eclipsed. All the methods of determining stellar radii dis-

cussed in this section require a knowledge of limb darkening, and it has been found more convenient to assume plausible values for the limb-darkening from the theory of stellar atmospheres, than to attempt to determine the limb-darkening and radius of a star simultaneously. Although such a simultaneous determination is possible in principle from the light curve of an eclipsing binary, it is usually found to be very difficult in practice. If only shallow partial eclipses are observed, the separation of the two variables is very difficult. The choice of suitable limb-darkening coefficients is discussed in Chapter 7. Fortunately, if the eclipses are deep and total, the values obtained for the radii are fairly insensitive to the value assumed for the limb darkening, but to determine the semi-axis of the orbit accurately, accurate measures of both spectra are needed, and these usually cannot be obtained for systems showing deep eclipses, because they are the systems with components of very unequal luminosities. Thus the conditions for good determinations of fractional and absolute radii contradict each other, for systems in which two spectra are easily observed must contain stars of nearly equal luminosity, and probably of nearly equal radii. They will show shallow eclipses, and are more likely to show only partial eclipses than are systems in which one star is appreciably larger than the other.

Thus the compilation of a list of reliable radii of stars is also difficult, though not as difficult as the preparation of a list of reliable masses. It is hoped in the near future that a group of specialists will be able to work on the preparation of a catalogue of absolute dimensions determined from eclipsing stars. Popper has determined radii for most of the stars in his lists of well-determined masses (1967b, 1970), and the values of radii determined interferometrically are given in the publications already cited.

Stellar radii can be determined indirectly for any star for which the total luminosity and effective temperature are known. The scale of effective temperature, however, depends, at least partly, on data obtained from eclipsing binaries (Kuiper, 1938a). Such determinations, therefore, must always be of secondary quality.

Chapter 6

APSIDAL MOTION

CAUSES OF APSIDAL MOTION

The longitude of periastron of a binary orbit, denoted by ω, defines the direction of the line of apsides (i.e. the major axis extended indefinitely in either direction) in the orbital plane. It is an element of the orbit, and under the assumptions used in deriving the equations of elliptical motion of two gravitating bodies, it is constant. Those assumptions are that the bodies can be regarded as mass points, that they move in accordance with Newton's law of gravitation, and that the two bodies form a gravitationally isolated system. If any of these three assumptions fail, the element ω does not remain constant.

In close binary systems, in which the separation of the stars is often less than 10 times their radii, it may well seem that it is no longer justifiable to treat the component stars as mass points. Newton showed, in the *Principia* (1686), that a homogeneous spherical star still acts gravitationally as a mass point, as also does one built up of homogeneous spherical shells so that its density at any point is a function of the distance from the centre of the star only. Two stars very close to each other will deform each other, however, and destroy the spherical symmetry. The assumption that the stars are mass points is then insufficient for a complete description of the motions of the two stars. The most readily detected departure of the observed motion from the predictions of the simple theory is a steady increase in the value of ω, which is referred to as *advance* or *rotation* of the line of apsides. The derivation of this result is given by Cowling (1938), Sterne (1939a), and Kopal (1959d). It is not a difficult derivation, but takes up a lot of space and it is not repeated in full here. The distortion of the two stars that arises both from their mutual tidal distortion and from their own rotations is first expressed in a series of surface harmonics.

The external gravitational potential of each distorted star can be expressed in a corresponding series of Legendre polynomials. Of these, the second order polynomials are the most important terms. From them it can be shown that the potential of the distorted star 1 at the mass centre of star 2 is given by

$$r_1^5 k_{12}\left\{\frac{Gm_2}{R^6}+\frac{\omega_1^2}{3R^3}\right\}. \qquad (1)$$

In this expression, G is the gravitational constant and R is the instantaneous distance between the two stars. The first term in the bracket arises only from the tidal distortion and the second only from the rotational distortion (ω_1 is the angular velocity of rotation of star 1). The potential of star 2 at the mass centre of star 1 is obtained by exchanging all subscripts, except that k_{22} is written for k_{12}. The two constants k_{12} and k_{22} are given by

$$k_{i2}=\frac{3-\eta_2(r_i)}{4+2\eta_2(r_i)}$$

where $\eta_2(r_i)$ is the value of the variable η_2 that is zero at $r = 0$ and satisfies the differential equation

$$r\frac{d\eta_2}{dr}+\eta_2^2-\eta_2-6+\frac{6\varrho}{\bar{\varrho}}(\eta_2+1)=0,$$

in which the density ϱ is a function of r, $\bar{\varrho}$ is the mean density of the star, and r varies from zero to r_i. The parameters k_{12} express the influence of the internal structure on the disturbing potential of the stars, since $k_{i2} = 0$ if the stars are mass points, and $k_{i2} = 0.75$ if they are homogeneous. The variation in ω produced by the disturbing potential (1) is proportional to its partial derivative with respect to the eccentricity, and the observed variation in ω is the sum of the variations produced by each component. The final result is that the rate of apsidal advance, $\Delta\omega$ per orbital revolution, is given by

$$\frac{\Delta\omega}{2\pi}=k_{12}\left[\frac{m_2}{m_1}\{15f_2(e)+g_2(e)\}+g_2(e)\right]r_1^5+k_{22}\left[\frac{m_1}{m_2}\{15f_2(e)+g_2(e)\}+g_2(e)\right]r_2^5 \qquad (2)$$

where

$$f_2(e)=(1-e^2)^{-5}(1+3e^2/2+e^4/8)$$
$$g_2(e)=(1-e^2)^{-2}.$$

The terms in $f_2(e)$ arise from the tidal distortion of each star, those in $g_2(e)$ from the rotational distortion. It has been assumed, to reach equation (2) that the two components of the system rotate in synchronism with the orbital period. If this is not so, the terms in $g_2(e)$ must be multiplied by the square of the ratio of the actual rotational velocity to that corresponding to synchronism. Note that if e is small enough for its square to be neglected, both $f_2(e)$ and $g_2(e)$ are unity, and equation (2) can be simplified. Most systems in which apsidal motion is well established, however, have an appreciable orbital eccentricity. Because the observed motion of the apse is the sum of that produced by both stars, the quantities k_{i2} cannot be determined separately. Only a weighted mean

$$\bar{k}_2=\frac{c_1k_{12}+c_2k_{22}}{c_1+c_2}$$

can be determined, where c_1 and c_2 are the coefficients of k_{12} and k_{22} in equation (2). If the two stars have equal masses and equal radii $c_1 = c_2$ and, probably, also $k_{12} = k_{22}$.

Equation (2) is actually derived by considering the variation of $\tilde{\omega} = \omega+\Omega$. Since no forces are involved that can exert a couple on the orbital plane, the angle Ω should remain constant and it is legitimate to replace the variation $\Delta\tilde{\omega}$ by $\Delta\omega$. The quantity $\bar{k}_2$ is only a crude measure of the central condensation of the two stars. If the internal structure of stars can be well represented by a family of models (e.g. polytropes) then $\bar{k}_2$ can be directly related to the ratio $\varrho_c/\bar{\varrho}$ (where ϱ_c is the central density). It is better, however, to determine $\bar{k}_2$ from the observations and to compare it with calculated values of $\bar{k}_2$ for various models. A fine choice between slightly different models is not possible, but any model that gives a value of k_{i2} widely different from the observed value can be rejected. It is possible, at least in principle, to distinguish between models appropriate to different stages of stellar evolution.

If the period of apsidal rotation is denoted by U, then

$$P/U = \Delta\omega/2\pi$$

and

$$\bar{k}_2 = \frac{P/U}{c_1+c_2}.$$

The computed values of $\bar{k}_2$ for stellar models are approximately 10^{-2}. For systems containing stars of comparable mass (which are those for which m_1/m_2, r_1, and r_2 are most likely to be well determined) both m_1/m_2 and m_2/m_1 are of order unity. The coefficients c_i, therefore, are of the order $10r_i^5$. A typical value of r_i is approximately 0.2, and therefore a typical value of c_1+c_2 is between 10^{-2} and 10^{-3}. Thus the expected value of U is 10^4 to 10^5 times the orbital period, P. The expected values of U for eclipsing binaries with periods of a few days are in the range 100 to 1000 years. Shorter apsidal periods may be encountered in systems containing rapidly rotating stars, and U (or $\bar{k}_2$, whichever is regarded as the unknown) is very sensitive to the values of the fractional radii of the two stars r_1 and r_2. Very close systems may show much more rapid apsidal advance. Fractional radii are often not well determined. Although the formal uncertainties may be quite small, the true uncertainties in the radii may be much greater because of assumptions made (e.g. about the limb darkening) in the solution of the light curve. An uncertainty of 5 per cent in the fractional radii can lead to an uncertainty of up to 25 per cent in the value of $\bar{k}_2$. The inevitable uncertainties in m_2/m_1 and in the rotational velocities of the stars are not important compared with those in the fractional radii.

The second assumption that must be examined in the discussion of the variation of ω is that the two components of a close binary system revolve around their centre of mass in conformity with Newton's law of gravitation. Einstein's theory of relativity predicts an advance of the line of apsides, even if the two stars can be considered as mass points. The formula is a standard one in texts on relativity mechanics, and again the derivation is not given here. Kopal (1965) has given the formula in the form most useful for the present discussion, namely

$$U'/P = 1.57\times10^5\frac{A(1-e^2)}{m_1+m_2}$$

where U' is the period of relativistic apsidal advance, and A is the major semi-axis of the relative orbit of the two stars (expressed in units of the solar radius) and m_1 and m_2 are, as usual, in solar masses. The whole factor $A(1-e^2)/(m_1+m_2)$ is likely to be of order unity in systems containing massive stars, and in such systems the period of relativistic apsidal advance is only about 10 times as long as the period of apsidal advance that results from the mutual distortion of the two stars. Thus the apse advances more quickly, by an appreciable amount, than it would if there were no relativistic effect. If the relativistic effect is neglected in such systems, the value deduced for U is too short by an amount of the order of 10 per cent, and too large a value is deduced for $\bar{k}_2$. Except in the most massive systems, this effect is unlikely to be important, especially if the radii r_1 and r_2 are not well determined.

The third condition for a constant value of ω that may not be fulfilled is that the binary system should be a gravitationally isolated system. In Chapter 3 it is shown that many binary systems are members of triple and multiple systems. The presence of a third body perturbs the orbit of the close pair, and one of the most sensitive elements is the longitude of periastron, ω. The motion of the apse in a triple system is a difficult problem in celestial mechanics that has been investigated by Slavenas (1927), Lyttleton (1934), Brown (1936, 1937), Martynov (1948), and Kopal (1959d, 1967). Some special cases of the general theory have been investigated for the purpose of understanding the motions of artificial satellites (Eckstein *et al.*, 1966 a, b). All investigators have started from some form of the lunar theory because the stellar problem of three bodies is formally very similar to the lunar problem. Most triple systems, as is shown in Chapter 3, consist of a close pair of stars accompanied by a relatively distant companion. The chief difference between the stellar and the lunar problems is that in the former all three masses are comparable.

Slavenas considered the simple case of two coplanar orbits, and supposed the orbit of the third body to be circular. He found that the disturbing body causes the line of apsides of the orbit of the close pair to advance steadily, and the apsidal period, U'', is given in terms of the orbital periods, P and P', of the close and wide systems respectively, by

$$P'/U'' = \frac{0.75m_1}{m_1+m_2+m_3}(P/P') + \text{higher terms.} \tag{3}$$

Now P/P' is of the order 10^{-2} or 10^{-3} (Chapter 3), and since $m_1/(m_1+m_2+m_3)$ is clearly of the order 1/3, it follows that

$$P'/U'' \approx 0.25\times10^{-2} \quad \text{or} \quad 10^{-3},$$

or

$$P/U'' \approx 10^{-4} \quad \text{to} \quad 10^{-6}.$$

That is, the apsidal advance produced by the presence of a third body may well be comparable in rate with the apsidal advance arising from other causes.

Lyttleton and Brown have confirmed Slavenas' principal result. Lyttleton showed that if the two orbital planes are inclined at a small angle, only the higher terms in equation (3) contain this relative inclination. Brown generalized Lyttleton's result to higher relative inclinations and also took account of the eccentricity of the orbit of the third body. The effect of this is to reduce the coefficient of the leading term of equation (3)—that is, slightly to slow down the rate of apsidal motion—and further to modify the higher terms. The formulae given by Lyttleton and Brown express the change in the angle $\tilde{\omega} = \omega + \Omega$, where Ω refers to the line of nodes of the two orbital planes, and *not* to that of either of them with the sky. Lyttleton found that the line of nodes *recedes* at approximately the same rate as the apsidal line advances. The angle between the two lines therefore increases at twice the rate given by equation (3), but that equation gives a good approximation to the motion of the apse relative to a fixed direction in space. The angle ω in these investigations is measured from the line of nodes of the two orbital planes. Observed values of ω are measured from the (usually indeterminate) line of nodes of the orbital plane of the close

pair and the tangent plane to the sky (Fig. 6.1). Thus the observed changes in ω depend not only on change of periastron in the orbital plane, but on the orientation of, and changes in, that plane with respect to the observer. Information on these matters is scanty, as shown in

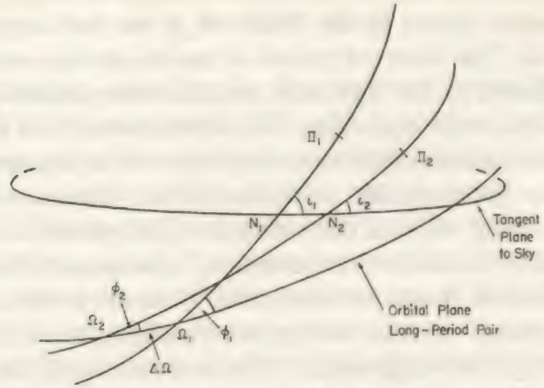

FIG. 6.1. Distinction between the observational and theoretical significance of ω for a spectroscopic pair belonging to a triple system. Two positions $\Omega_1\Pi_1$ and $\Omega_2\Pi_2$ are shown for the orbit of the close pair. The angle referred to as ω in theoretical work is represented by the arc $\Omega\Pi$ (Π denotes periastron). The observed value of ω is represented by the arc $N\Pi$. Note that $N_1\Pi_1 - N_2\Pi_2$ is not necessarily equal to $\Omega_1\Pi_1 - \Omega_2\Pi_2$, but depends on it through values of Ω_1, $\Delta\Omega$, i_1 and i_2.

Chapter 3, and the observable changes in ω are unpredictable for any particular triple system.

Brown made some numerical comparison of his predictions with the observations of ξ Ursae Majoris, in which the inclination of the two orbital planes is about 40° (or 140°). His method is not suited for a discussion of general results, except for small relative inclinations of the orbital planes. Kopal (1959d, 1967) tackled the general problem in a different way. In the earlier version, the angles ω and $\tilde{\omega}$ were confused. His later, corrected formulae reduce to Lyttleton's as a limit for small values of the relative inclination. The formula for the increase in ω per revolution contains a term in ω. This term is small compared with the secular term if the two orbits are nearly coplanar, but it becomes dominant at a value of the relative inclination deter-

mined by the three masses and two orbital inclinations (Eckstein *et al.*, 1966b). The line of apsides may then oscillate with a period comparable with the period of apsidal advance in the coplanar case. In a triple system, therefore, the line of apsides of a close pair may *advance* or *regress*, because of a third body, at a rate comparable with that of the apsidal advance caused by the distortion of the two components of the close pair. The observed period of apsidal motion may be made longer or shorter by the third body, and the value deduced for the $\bar{k}_2$ may thus be lower or higher than the correct value. If the majority of triple systems contain coplanar orbits, values of $\bar{k}_2$ determined from them will be systematically overestimated. As shown in Chapter 3, there is no strong evidence that orbits in triple systems are coplanar. If high relative inclinations are common, some cases of (temporary) apsidal regression should be observable. None are known.

Binary systems are also not gravitationally isolated if they are surrounded by a resisting medium. The resistance itself does not have any secular effect on the line of apsides (Smart 1953), but Hadjidemetriou (1967) has pointed out that the mutual gravitational attraction of the two stars is modified by the attraction of the matter between them, and, if the orbit is elliptical, this varies as the stars revolve around each other. He finds that this produces a slow secular *recession* in the line of apsides given by

$$\Delta\omega = \frac{-4\pi G}{c^2} r^4\sigma(r) \sin^2 v \, dv,$$

where G is the gravitational constant, c the areal constant, r the distance between the stars, $\sigma(r)$ the density of the medium, and v the true anomaly. If $\sigma(r)$ has a constant value σ, and if e is small, the formula becomes

$$\Delta\omega = -GP^2\sigma + \text{terms in } e^2 \text{ and higher powers.}$$

If U^* is the period of apsidal motion from this cause

$$P/U^* = -GP^2\sigma/2\pi + \text{higher terms.}$$

The minus sign is merely a formal indication that the motion is a regression, and it can be dropped. If the numerical value of G (6.67×10^{-8} dynes/cm^2) is substituted, and a typical value of 10^5 seconds is assumed for P, then

$$\sigma \approx 10^{-2}P/U^* \quad \text{g/cm}^3.$$

For apsidal motion caused by the distortion and internal structure of the stars, P/U is approximately 10^{-4} to 10^{-5}. A rate of apsidal regress of even 1 per cent of that of the advance from this latter cause, therefore, requires a circumstellar medium with a density of 10^{-8} or 10^{-9} g/cm^3. If the medium were pure hydrogen, this would correspond to 10^{16} or 10^{15} atoms/cm^3. This is much denser than the average values of circumstellar matter in binaries that are given in Table 12 (Chapter 9). Secular motion of the line of apsides caused by a circumstellar medium therefore seems to be observationally unimportant.

OBSERVATIONAL DETECTION OF APSIDAL MOTION

The longitude of periastron (referred to as the node of the orbital plane and the tangent plane of the sky) can be determined, in principle, by either spectroscopic or photometric observations. It is shown in Chapter 1 that spectroscopic determinations of ω are not to be trusted, at least for eclipsing binaries. A long series of spectroscopic observations of the system AR Cassiopeiae showed evidence for an apparent apsidal motion with a period of 400 years. The limited photometric evidence never clearly supported this, and eventually, further spectroscopic evidence showed that there is not a steady advance of the line of apsides. It is possible that in non-eclipsing systems, the velocity curves are subject to less distortion and the longitudes of periastron may be more safely derived from them. For example, the system H. R. 8800 shows very convincing evidence of apsidal motion in a period of 156 years (Petrie and Petrie, 1967). If the determination of the apsidal-motion constant $\bar{k}_2$ is to be free of any assumptions, however, it is

essential that only eclipsing binaries are used, and the results from these should be at least partly based on photometric observation.

As explained in Chapter 1, the longitude of periastron can, in principle, be determined directly from the light curve, if both eclipses have been completely observed, although, in practice, its determination in this way is often difficult. It is shown in Chapter 4, however, that the interval between successive minima of an eclipsing binary is changed by rotation of the line of apsides. The careful observation of such changes is probably the best way of studying apsidal motion. Kopal (1965) has objected to this procedure because of his earlier finding (1959d) that there are many other potential causes of apparent period variation in eclipsing systems. As is remarked in Chapter 4, however, Plavec (1960a) found that these other period variations are seldom likely to be detectable. Both minima should be studied, however, in order to eliminate the possibility that the apparent period changes are produced by orbital motion of the close pair around a third body. Observation of the secondary minimum may help to detect apsidal motion even before any apparent change in the orbital period has been detected, because it may be impossible to satisfy both minima with the same value of the period. Such a clue was used by O'Connell (1968) in the discovery of apsidal motion in the system HH Carinae.

The following section of this chapter contains a critical assessment of existing determinations of the apsidal motion constant $\bar{k}_2$. In the remainder of this section, criteria that can be used in such an assessment are considered. Equation (2) shows that the quantities needed to obtain a value for $\bar{k}_2$ are: the mass ratio of the system, the fractional radii of the two component stars, the period of the system, and the period of apsidal rotation.

The following criteria that observations of a given system must meet may therefore be set up.

(i) Spectroscopic observations must be available, and preferably both spectra should have been observed.

(ii) Accurate times of minima should be available over an interval of at least one apsidal period. It is highly desirable that both minima should have been observed and timed.

(iii) In order that the regular variation in times of minima should be distinguishable from random variations from other causes, the orbital eccentricity must not be too small. In practice, it is reasonable to require that eP/π be not much less than 0.01.

(iv) At least one good complete light curve should be available from which reliable fractional radii can be determined. Because of the extreme sensitivity of $\bar{k}_2$ to these radii, this is the most important criterion. It is also the hardest to apply in practice, because published uncertainties in radii are often misleadingly small. I have applied the criterion by requiring that the photometric elements of a system should be given quality "*C*", or better, in the catalogue published by Koch *et al.* (1970).

Although observations of both spectra are desirable, in order to obtain the mass ratio directly, some relaxation of criterion (i) can be allowed in circumstances in which an indirect estimate of the mass ratio is possible. Similarly, although it is desirable that a large number of both minima have been timed, if only a few secondary minima have been observed, or if more than one spectroscopic determination of ω is available, then it may be possible to conclude safely that the observed period variation is caused by apsidal rotation. It should be regarded as essential, however, that some photometric evidence is available. The apsidal period can be quite accurately determined from observations that extend over only a short interval (De Kort, 1956) but it is prudent to require that at least one apsidal period has been observed before a determination of $\bar{k}_2$ is accepted as reliable, in order to be sure that the observed period variation is periodic.

Observations of a system that meet all these criteria should provide first-class values of $\bar{k}_2$. If the observations fail to meet any of criteria (i)–(iii) they should be suitable for second-class determinations. If they fail to meet criterion (iv), or any two of (i)–(iii), they may be accepted for third-class determinations. Observations of lower quality than this are not suitable for a determination of $\bar{k}_2$.

CRITICAL ASSESSMENT OF DETERMINATIONS OF $\bar{k}_2$

Several compilations of observational values of $\bar{k}_2$ have been published since Sterne's pioneering discussion (1939b). The most recent are those by Kopal (1965) and Semeniuk (1968b). In this section, all those systems discussed by either Kopal or Semeniuk (except DI Herculis for which only relativistic apsidal motion was found to be important) are reviewed in order to evaluate critically the degree of reliability of the estimate of $\bar{k}_2$. A few other systems, for which information has become available since Semeniuk's paper was published, are also included in the discussion.

The systems from which useful values of $\bar{k}_2$ can be derived are listed in Table 9. They are divided into three classes in accordance with the criteria set up in the previous section, which were applied in the manner outlined there. Table 9 includes all the systems considered by Semeniuk to give reliable values of $\bar{k}_2$, but does not include all the systems listed by Kopal. The values of $\bar{k}_2$ have been calculated afresh, for all but V477 Cygni and the last three systems, from the set of elements given for each system in the catalogue by Koch *et al.* (1970)—from which the "orbit grade" has also been taken. I have followed Kopal in assuming that the rotational velocity of the components is synchronized with their maximum orbital velocity, unless there is evidence that it is otherwise. The contribution of rotational distortion to the apsidal motion is never important, and the value derived for $\bar{k}_2$ is not sensitive to this assumption. I have also followed Kopal in correcting the observed apsidal period of Y Cygni from $5600P$ to $5900P$ in order to take account of the expected relativistic component and in assuming his estimated mass ratios for those systems for which direct determinations are not available.

Although Y Cygni fulfils all the criteria for a first-class determination of $\bar{k}_2$, there is some uncertainty in the final result because of the possibility that there is a third body in the system. Two sets of orbital elements obtained at Victoria have values of V_0 that differ by 10 ± 3 km/sec (Redman, 1931). The spectrum is very difficult to measure,

TABLE 9. OBSERVATIONAL DETERMINATIONS OF $\bar{k}_2$

System	Spectral type	Period (days)	U/P	Interval observed (unit: U)	No. of spectra observed	eP/π (days)	Orbit grade	$\bar{k}_2$
First class								
Y Cyg	O9.5+O9.5	3.00	5900[1]	1.00	2	0.13	C	0.0080
CO Lac	B9+B9	1.54	10,000	1.72	2	0.01	*B*	0.0040
Second class								
GL Car	B3+B4	2.42	3800	1.84	0	0.12	*B*	0.0129
AG Per	B5+B7	2.03	13,750	~0.5	2	0.05	C	0.0061
Third class								
V477 Cyg	A3+F2	2.35	54,300	0.18	2	0.22	C	0.0062
RU Mon	B9	3.58	29,900	0.47	1	0.04	C	0.0090
YY Sgr	A0	2.63	~46,700	~0.2	0	0.13	C	0.0043
V526 Sgr	A0	1.92	27,800	0.47	0	0.14	*B*	0.0040
W Del	A0+G5	4.81	3900	1.18	1	0.06	C	0.0038[2]
β Per	B8	2.87	4070	5.90	1	0.008	C	0.0036[2]
TX UMa	B9+F	3.06	4100	1.44	1	0.02	C	0.0031[2]

[1] Corrected for relativistic term.
[2] k_{22} for secondary components only, see text.

10*

and the difference is barely significant. There is, however, a need for caution in the interpretation of the apsidal motion. The system of CO Lacertae now qualifies for inclusion among the first-class determinations because of spectroscopic observations made by Smak (1967) and a new light curve obtained and analysed by Semeniuk (1967).

Only the lack of spectroscopic observations prevent GL Carinae from being suitable for a first-class determination of $\bar{k}_2$. There can be little reasonable doubt that the mass ratio is close enough to unity for the derived value of $\bar{k}_2$ to be quite insensitive to the precise value assumed. Many, therefore, may be content to regard the value of $\bar{k}_2$ as fully reliable: I prefer to err on the side of caution. Spectrograms of the star are being obtained, and it is only a matter of time until all criteria are met. Some recent determinations of times of minima, however, raise the question whether their variation is fully explained by apsidal motion (Quast, 1969). Similarly, it can be hoped, that time and assiduous observations of minima will promote AG Persei to the first-class section. The visual companion of this star is unlikely to have any effect on the apsidal motion, even if it is physically connected with the close pair.

The system of V477 Cygni has presented problems. The most thorough discussion of the light curve and of apsidal motion has been published by O'Connell (1970). The information in Table 9 is based on his work, although the orbit grade "*C*" is applied by Koch, Plavec, and Wood to an earlier study by Rodonò (1967). The elements he derives are different from those obtained by O'Connell, probably because Rodonò has assumed a smaller value of the orbital eccentricity. O'Connell's argument that the eccentricity must be about 0.30 to account for the observed displacement of times of minima appears to be conclusive. Moreover, his elements lead to a value of $\bar{k}_2$ more in accord with expectations than the value of 0.046 that I obtain from Rodonò's. (The value of $\bar{k}_2$ published by Rodonò appears to be affected by numerical slips in the computation.) The situation is complicated by a disagreement between the spectroscopic results obtained by Pearce (1958) and Popper (1968), although, again, the more recently published results appear to be preferable. Although formally the

system meets the criteria for a second-class determination, and the result probably should be regarded as one, I have again preferred to err on the side of caution.

The agreement between the observed and expected values of $\bar{k}_2$ for RU Monocerotis has been greatly improved by the revised value of U/P obtained by Martynov (1965). Only one spectrum has been observed, however, and the photometric observations still cover only a small portion of the supposed apsidal period. Prikhodko (1961) reported an abrupt period change in the system. The apsidal constant deduced must, therefore, be regarded as one of the least reliable in the Table. For the two systems YY Sagittarii (Keller and Limber, 1951) and V526 Sagittarii (O'Connell, 1967), there is no reason to doubt the reality of the apsidal motion, but more observations, both photometric and spectroscopic, are needed to confirm the values of $\bar{k}_2$.

The group of three systems, W Delphini, β Persei, and TX Ursae Majoris, is in a different category from the others. These are all semi-detached systems. Eclipses of the secondary components are shallow and difficult to time accurately: these components are subgiants, and their spectra are visible, if at all, only during the primary eclipses. Thus it is hard to establish that an observed period variation is caused by apsidal motion, and almost impossible to obtain a definitive value for $\bar{k}_2$. Plavec (1960b) has assumed that the systems do exhibit apsidal rotation, and that the values of k_{21} for the primary components are those computed for main-sequence stars of the same masses. Because the subgiant components are large compared with the primaries, the value obtained for k_{22} is not very sensitive to that assumed for k_{21}. The uncertainty in the mass ratio of these systems is probably more important. The close agreement between the three values of k_{22} strengthens confidence in the interpretation of the observed period changes as due to apsidal motion. Confidence is increased in the case of β Persei by a recent rediscussion of the available spectroscopic evidence (Hill *et al.*, 1971). For TX Ursae Majoris, Grewing and Herczeg (1966) have found that the few recorded times of secondary minimum are not in conflict with the hypothesis of apsidal motion. More information could be obtained for these systems if secondary

eclipses were observed in the infra-red, where they are deeper, as Chen and Reuning (1966) have done for β Persei.

Table 10 contains a number of systems that are worth further observation. In most of them, apsidal motion can be regarded as established, but observations are insufficient for useful determinations

TABLE 10. SYSTEMS WORTH FURTHER OBSERVATION

System	Period (days)	U/P	Criteria not met	References
V889 Aql	11.12		(i) (ii) (iv)	Semeniuk (1968b)
Y Cam	3.31		(i) (ii) (iii) (iv)	Plavec *et al.* (1961)
HH Car	3.23	75,000	(i) (ii) (iv)	O'Connell (1968)
V346 Cen	6.32	11,000	(i) (ii) (iv)	Dugan and Wright (1939)
CW Cep	2.73	~5000	(ii) (iv)	Koch and Nha (private comm.)
SY Cyg	6.01	~2000	(i) (ii) (iv)	Wachmann (1961)
V380 Cyg	12.43	59,300	(ii) (iv)	Semeniuk (1968b)
UW Lac	5.29	11,600	(i) (ii) (iv)	Strohmeier (1961)
GN Nor	5.70	31,000	(i) (ii) (iv)	de Kort (1954)
V523 Sgr	2.32	32,900	(i) (ii) (iv)	de Kort (1942)
AO Vel	1.6	~11,500	(i) (ii) (iv)	Oosterhoff and van Houten (1959)
DR Vul	2.25	6200	(i) (ii) (iv)	Semeniuk (1968b)
HBV 242	6.26	11,000	(i) (ii) (iv)	Wachmann (1961)
HV 7498	3.47	59,600	(i) (ii) (iv)	Sterne (1939b)

of $\bar{k}_2$ to be made. The apparently periodic variation in the times of minima of Y Camelopardalis is probably not due to apsidal motion (Plavec *et al.*, 1961). The rapid apsidal motion discovered in CW Cephei by Koch and Nha (unpublished) is of interest, because both spectra are visible: the system may eventually yield a good value of $\bar{k}_2$, but a better analysis (being undertaken by Nha) is needed. The primary component of V380 Cygni is an early-type giant. Luyten (1938) was the first to suggest that the system shows apsidal motion: he estimated $U = 280$ years. Popper (1949) found that such rapid apsidal motion was unlikely, and this conclusion was confirmed by Batten (1962)

who found $U \geq 2000$ years. From new observations of times of minima, Semeniuk has independently suggested that $U = 2019$ years. This close agreement is no doubt fortuitous, but both spectra are visible and the system may eventually yield a useful determination of $\bar{k}_2$ for a star not on the main sequence.

TABLE 11. SYSTEMS IN WHICH APSIDAL MOTION IS UNLIKELY TO BE ESTABLISHED

AO Cas	XX Cep
AR Cas	MR Cyg
RZ Cas	δ Ori
RS CVn	β Sco
	α Vir

In Table 11 a number of systems are presented which have been supposed to show apsidal motion, and for many of which values of $\bar{k}_2$ have been published. They are, however, probably unsuitable for reliable determinations of $\bar{k}_2$. The spectra of the secondary components of AO Cassiopeiae and β Scorpii are believed to be badly contaminated by the spectra of gaseous streams (Abhyankar, 1959), and the latter system shows no eclipses. The evidence for apsidal motion in AR Cassiopeiae depends only on spectroscopic observations (Petrie, 1946). It is supported neither by photometric determinations of ω nor by Petrie's later unpublished spectroscopic observations. Parenago (1952) has observed RZ Cassiopeiae for three of its supposed apsidal cycles, and he could find no periodic variation in the times of minima of amplitude greater than $0^d.004$. Plavec (1960b) first studied RS Canum Venaticorum together with the other three systems containing subgiants, that are listed in Table 9. The value of k_{22} is very much less than that found for those three systems, and Plavec later expressed doubt about his original conclusion (Plavec *et al.*, 1961). These doubts are increased by the discovery of a variable distortion in the light curve (Catalano and Rodonò, 1967, 1968) which is discussed in more detail in Chapter 8. At least one photoelectric light curve of XX Cephei has a very distorted secondary minimum (Fresa, 1953) which may account

for the apparent changes in phase of that minimum. Only one-fifth of the supposed apsidal period has been observed. A new photoelectric light curve of MR Cygni (Hardie and Hall, 1969) makes it appear unlikely that apsidal motion has yet been detected in this system. The spectral type of the primary star was misprinted in an early paper on this system. The true type is B5, or even earlier, and not A0. Even if Kopal's (1965) value of $\bar{k}_2$ is accepted, this change in spectral type would modify the comparison between theory and observation for the system. The eclipses of δ Orionis and α Virginis are both very shallow, those of the latter may not exist at all. The system δ Orionis is unlikely to yield good values of the radii and therefore any determination of $\bar{k}_2$ must be very uncertain. There is only spectroscopic evidence for apsidal motion in α Virginis, and the spectrum is known to be unusual (Struve *et al.*, 1958a).

COMPARISON WITH THEORY

Theoretical values of $\bar{k}_2$ can be computed for any stellar model for which the density distribution is known. The earliest attempted comparison of theory and observation that used modern stellar models was made by Schwarzschild (1958) with his own models and those of Kushwaha (1957). Kopal's comparison (1965) made use of the same models. Both investigators found that empirically derived values of $\bar{k}_2$ are too small especially for stars of late B or early A spectral types, for which the discrepancy seemed to be a factor of three. A more detailed comparison was made by Semeniuk and Paczynski (1968) who computed new models in the range $4m_\odot$ to $16m_\odot$ specifically for this comparison, using modern values for the opacities and energy-generation rates. The initial main sequences (defined by the theoretical relations between $\bar{k}_2$ and effective temperature) used by Kopal and Schwarzschild, on the one hand, and by Semeniuk and Paczynski on the other are shown in Fig. 6.2. The coordinate axes are so arranged that the luminous end of the main sequence is to the left of the figure, while evolution (i.e. decreasing $\bar{k}_2$, or increasing central condensation) proceeds upwards and to the right, just as in a conventional Hertzsprung–Russell diagram. Semeniuk and Paczynski calculated models for three different assumed chemical compositions, and Fig. 6.2 shows that the relation between $\bar{k}_2$ and effective temperature is not very sensitive to the chemical composition. It also shows that a large part of the systematic difference between theory and observation found by Kopal and Schwarzschild was caused by the theoretical models they used.

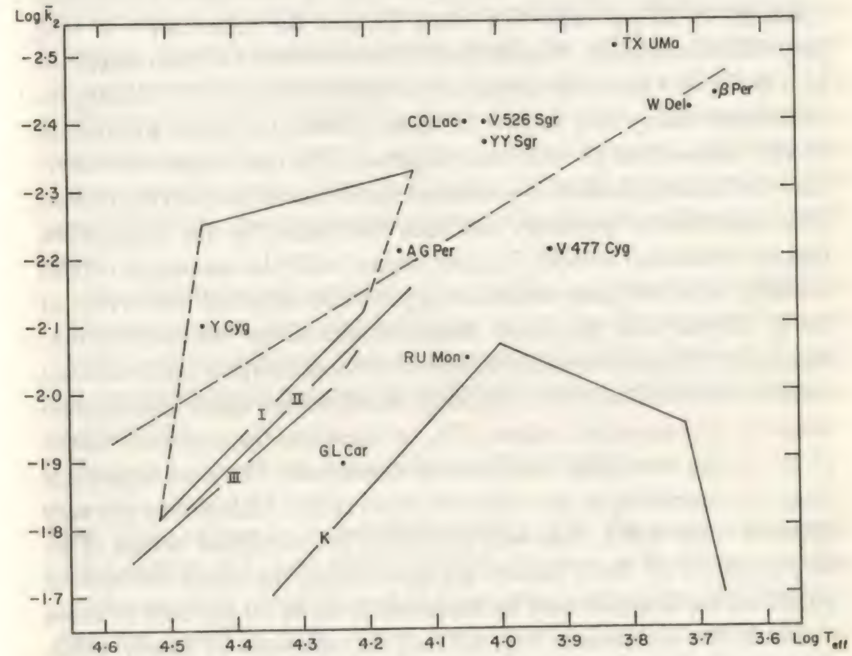

FIG. 6.2. Plot of log $\bar{k}_2$ against log T_{eff} for the eleven systems listed in Table 9. The line labelled *K* is the initial main-sequence relation adopted by Kopal. The lines labelled I, II, III are initial main-sequence relations computed for three different chemical compositions by Semeniuk and Paczynski (1968). The ends of these lines correspond to stars of $16m_\odot$ (left) and $4m_\odot$ (right). The effect of evolution is indicated by the trapezoidal figure constructed on line I. The upper boundary of this figure corresponds to the reduction of core hydrogen from 60% to 10%, which takes 6 million years for the star of $16m_\odot$ and nearly 70 million for the star of $4m_\odot$. The dashed line indicates the mean relation for all the observed points, except the three subgiants.

A useful comparison between theory and observation must include some attempt to estimate the uncertainty of the empirical data. This is very difficult because of the many sources of uncertainty and because of real doubt about their relative contributions. The most important source, because of the sensitivity of $\bar{k}_2$ to their values, is that in the fractional radii. The requirement that the photometric elements used have been given grade "*C*" by Koch *et al.* (1970) serves to minimize the uncertainty in $\bar{k}_2$ from this source, but in some cases it may still reach 20 to 30 per cent. For some systems the uncertainty in U/P may still be important. Thus $\bar{k}_2$ for RU Monocerotis has been increased by a factor of 3 just from the use of an improved value of U. An appreciable uncertainty may remain for those systems for which a complete apsidal period has not yet been observed. The next important uncertainty is the contribution of rotational distortion to the total distortion. This contribution probably has been maximized by the assumption that the rotational velocity is synchronized with the maximum orbital velocity. Most of these orbits are appreciably elliptical and errors of up to 10 per cent may have been introduced by this assumption, except for Y Cygni for which there is a direct estimate of the rotational velocity. It is unlikely that rotational velocities are much greater than assumed, and empirical values of $\bar{k}_2$ are therefore the minimum values as far as the rotational distortion is concerned. The least important cause of uncertainty is the unknown mass ratios. Unless they are very different from unity, they will not affect the empirical values of $\bar{k}_2$ very much. If all these factors are considered, the worst determined points on the diagram may be uncertain in $\bar{k}_2$ by 50 per cent or more —that is, the ordinate in Fig. 6.2 may be uncertain by nearly ± 0.2, but most of them should be better than this.

The uncertainties in the abscissae should also be considered. Not only is there uncertainty in the identification of the temperature scale with spectral type, but some of the spectral types themselves are poorly determined. For instance, the primary component of AG Persei may be as early as B3, in which case it should be moved appreciably to the left in the diagram. The two components of V477 Cygni are of different spectral types. The position in Fig. 6.2 is a mean for

the two stars, weighted approximately according to their luminosity. The components of this system are less massive than any of the models computed by Semeniuk and Paczynski, and the detailed comparison of theory and observation for them should await the computation of $\bar{k}_2$ for a series of modern models of moderate mass. This probably also applies to RU Monocerotis whose secondary is almost certainly of later spectral type than the primary and the system should therefore appear to the right of its present position. The abscissae of the three subgiants are uncertain, because of the difficulty of observing their spectra. Semeniuk and Paczynski found the effective temperatures so uncertain that they preferred to compare the empirical values of $\bar{k}_2$ with the theoretical relation between $\bar{k}_2$ and mass. This comparison can only be made for three or four systems, however, and even the masses are not certain for all of these. Given the present state of the empirical data, the comparison made in Fig. 6.2 seems the most useful possible.

The elimination of unreliable data, and in two cases the availability of improved data, have also helped to reduce the discrepancy between theory and observation. The mean empirical relation between $\bar{k}_2$ and temperature for all the systems in Table 9, except the three subgiants, is given by

$$\log \bar{k}_2 = 0.578 \log T_{\text{eff}} - 4.581$$

and this is shown as a dashed line in Fig. 6.2. The group of systems CO Lacertae, YY, and V526 Sagittarii do seem to be more centrally condensed than expected, and Y Cygni also seems more condensed—but the possible third body may be affecting the value of $\bar{k}_2$. Whether or not GL Carinae is really less condensed than a main-sequence star must remain doubtful. As Semeniuk and Paczynski point out, the agreement of most of the empirical points with theory could be improved if the rotational contribution to the distortion were not included in the computations at all. On the other hand, the relation between $\bar{k}_2$ and effective temperature might be different if rotation were included in the stellar models. Semeniuk and Paczynski were careful to point out that this has not been done. They also suggest the

possibility that further changes in the assumed energy-generation rates and opacities could improve the agreement.

In view of the uncertainties remaining in both observations and theory, the agreement between them is probably as good as can be expected, especially from such fragmentary data. It is encouraging that improvements in both theory and observation have reduced the discrepancy, although most empirical values of $\bar{k}_2$ are smaller than the best available theory predicts. It is tempting to explain the remaining discrepancy as a result of evolution. There are, however, two arguments against this: (i) except for Y Cygni the stars concerned would have to be quite old to have evolved so far—in particular, AG Persei, which is a member of the II Persei, has not had sufficient time; (ii) the components of Y Cygni, CO Lacertae, and AG Persei lie close to the theoretical mass-radius relation for unevolved stars.

Table 10 contains one system that contains a giant star—V380 Cygni. Semeniuk (1968b) computed a provisional value of $k_{21} = 0.00026$. A model for a similar star computed by Semeniuk and Paczynski has $k_2 = 0.004$. The empirical value is very uncertain, however, because of the uncertainties in both the radii and the apsidal period. No real conflict between theory and observation exists for this star.

CHAPTER 7

CLOSE BINARY SYSTEMS AND STELLAR ATMOSPHERES

APPLICATIONS OF THE THEORY OF STELLAR ATMOSPHERES TO CLOSE BINARY SYSTEMS

The immediately preceding chapters are concerned with the derivation of fundamental data that are obtained largely or wholly from binary systems. The emphasis in this chapter is different. The theory of stellar atmospheres can stand very well by itself, and has been developed with little or no reference to the data obtained from binary stars. On the contrary, observers of binary stars have profited from studies of stellar atmospheres. The principal problem in the construction of a model atmosphere is to predict the continuous and line spectra of a star. Such predictions cannot easily be compared with observations of the spectra of binary stars, since the superposition of two spectra hinders the empirical determination of the intensity distribution in either of the component spectra. An eclipsing binary does, however, offer the opportunity to determine one quantity that can be found in no other way (except for the Sun), and that is the limb-darkening coefficient for the eclipsed star.

The surface intensity I_θ at any point on the apparent disc of a star is given, to a first approximation, by

$$I_\theta = I_0(1-u+u\cos\theta), \tag{1}$$

where I_0 is the central surface intensity, θ is the angle between the line of sight and the normal to the surface at the point in question, and u is the coefficient of limb darkening. By observation $u \approx 0.6$ for the Sun in visual light. Since this form of the law is linear in $\cos\theta$, it is commonly called the linear law. The actual variation of intensity over the disk of any real star (and of any realistic model) is more

complicated. It has long been recognized that limb darkening affects the shape of the light curve of an eclipsing binary, particularly near the contacts of an eclipse when the darkened regions of a stellar disk are being covered or uncovered. Thus, in principle, the coefficient u can be determined from the light curve, as is mentioned in Chapter 1. It is also mentioned in Chapter 5 that it is difficult to separate the effects of changing the limb-darkening coefficient from those of changing slightly the radii of the stars, particularly if the eclipses are shallow, although Linnell and Proctor (1970) find that it is nearly always possible to separate the two, given sufficient, accurate, and well-distributed observations. Limb-darkening coefficients are never likely to be known very accurately, however, and they are not particularly sensitive to the structure of a stellar atmosphere, or at least to that of model atmospheres. The coefficient u is to some extent similar to the apsidal motion constant $\bar{k}_2$ discussed in the last chapter: in both cases the observable quantity is difficult to determine, and furnishes only a rather crude discriminant between different possible structures. Grygar (1963) has suggested that it is more promising to compute limb-darkening coefficients from model-atmosphere data, and to use the value appropriate to the spectral type of the star being observed and the effective wavelength of the photometric observations, in the hope of obtaining more accurate values for the geometrical elements.

When observations of an eclipsing binary have been made in more than one colour, they can provide useful information on the variation of u with the wavelength, λ. A rather different variation is expected in the atmospheres of hot and cool stars. In the atmospheres of cool stars in which the opacity is caused chiefly by the negative hydrogen ion and molecular absorptions, u should decrease with increasing λ, while in the atmospheres of hot stars, if the dominant cause of opacity is electron scattering, u should be nearly independent of λ. It is found that the variation of u with λ can often be more accurately determined than can u itself. The limited data tend to support the hypothesis of electron scattering as the dominant cause of opacity in the atmospheres of hot stars, but observations of more suitable systems in several colours are still required.

Some attention has been paid to the question whether or not more accurate values of the geometrical elements of a system can be obtained by using a better approximation for the limb-darkening law. As early as 1956, Kopal and Shapley suggested that discrepancies between the geometrical elements derived for YZ Cassiopeiae from light curves in different colours could be removed if a term quadratic in cos θ were added to the limb-darkening law (1). Van't Veer (1960), however, found that the natural second approximation is a term cubic in cos θ, and he gave formulae for calculating the coefficients of both cos θ and $\cos^3 \theta$ if the surface temperature of the star and the wavelength of observation are known. Grygar's work (1963, 1965) on non-linear laws of limb darkening is based on Van't Veer's method, and he attempted to determine the cubic coefficient for the components of β Aurigae by comparison of the observations with predictions made from model atmospheres. He found that it was impossible to discriminate between different model atmospheres—at least for stars of spectral type A0—since a difference of less than 0.05 in the observed value of u cannot be considered physically significant, and the values predicted from different model atmospheres differ by quantities of this order. Grygar concluded that only in a very few cases would it be possible or useful to attempt to determine the cubic coefficient. This conclusion was confirmed by Cooper (1969) who was unable to find any differences between the geometric elements derived for *any* system with the alternative assumptions of linear and non-linear laws of limb darkening.

This result is not entirely surprising when it is recalled that in most systems the distribution of light over the surface of the stars is affected also by "reflection" and gravity darkening. "Reflection" is briefly discussed in Chapters 1 and 5. The geometrical problem of computing the light-intensity distribution over the surface of a star irradiated by a close companion of finite size is complicated but not insoluble. Kopal (1954) solved the problem analytically to a fairly high degree of approximation for spherical stars. With the help of a digital computer, however, it is now possible to do much better, and to allow for distortion (Hill and Hutchings, 1970). There is also, however, the

physical problem of computing how much of the incident radiation is actually reradiated. Eddington pointed out (1926b) that if a star is in radiative equilibrium, all the incident radiation must be returned to space. Many stars, however, have zones of convective equilibrium, and Ruciński (1969a) has recently estimated that less than half the incident radiation is reradiated by such a star in a close binary system. Napier and Ovenden (1970) suggest that mass motions generated by the absorbed energy will in turn affect the temperature distribution over the surface.

Gravity darkening will affect the observed light-intensity distribution because of the distortion of the star by tidal forces and by rotation. Von Zeipel's original deduction of the gravity-darkening theorem (1924) also applied to stars in radiative equilibrium, however, and Lucy (1967a) has investigated how the law should be modified for stars with convective zones. He found that the light intensity is much less sensitive to the surface gravity than it is for radiative equilibrium. His result is

$$T_{\text{eff}} \propto g^{0.08},$$

instead of

$$T_{\text{eff}} \propto g^{0.25}$$

found by von Zeipel for a radiative equilibrium. The symbol g denotes the acceleration due to gravity, and T_{eff} is related to the surface intensity, of course, by Stefan's law. Thus, any ellipticity effect observed in the light curves of systems containing stars with convective zones is caused almost entirely by the changing geometrical aspect of the distorted stars, and gravity darkening is a minor perturbation. If "reflection" must of necessity generate mass motions in the atmosphere of a "reflecting" star, Lucy's result may be of almost universal application to close binary systems.

Because it is very difficult to disentangle the effects of distortion, "reflection", and limb darkening by a conventional analysis of a light curve, and because large, fast digital computers are becoming increasingly available, a number of investigators are now attempting the construction of synthetic light curves. These are built from models into which the various effects can be inserted, and a given observed light curve can be compared with a series of computed light curves, each with its own specific values of radii, limb-darkening coefficients, orbital inclination, and effective temperatures. Papers have recently been published by Ruciński (1969 a, b), R.E. Wilson and Devinney (1971), D. B. Wood (1969), Cochran (1970), and Hill and Hutchings (1970), in which these methods are developed and applied. Hill and Hutchings have derived elements for the system of Algol by their method. One advantage of this method is that it cuts out the unsatisfactory process of rectification, which was an attempt to derive the "perturbations" of the light curve from the portion between eclipses, and the elements themselves from the eclipses. On the other hand, it is not evident that the solutions obtained are necessarily unique. This is not the time to attempt a definitive discussion of these new methods. Many of the papers referred to have only just appeared at the time of writing; some are still only in abstract form. It will be necessary to apply the methods more widely in order to discover any weaknesses they may contain, and they certainly must be expanded to include the effects of mass motions within the atmospheres and around the stars. There is a lively controversy amongst those working in the field as to whether it is better to build as complicated a model as possible, and include all the effects one can think of, or to build a very simple model and treat minor effects as perturbations (Gyldenkerne and West, 1970). There can be no doubt, however, that the development of these methods represents an important advance from the treatment of the solution of light curves as a geometrical problem to an attempt to take account of the real physical conditions within the stars. It is perhaps ironical that investigators in the field of close binaries are just beginning to make use of the theory of stellar atmospheres based on the assumption of local thermodynamic equilibrium at the time when students of stellar atmospheres are abandoning this hypothesis. It is inevitable, however. The fully developed theory must be used in this application, even if it is known to be inadequate in some respects. It is probable that model atmospheres based on this assumption will be sufficiently accurate for the purpose of obtaining

elements of eclipsing binaries. Continued cross-fertilization between the fields of close binary stars and stellar atmospheres is very desirable.

W URSAE MAJORIS SYSTEMS

The connection between stellar atmospheres and W Ursae Majoris systems may not be immediately obvious. In recent years these systems have been regarded as "contact systems" in the sense that both components fill their Roche lobes. This view has been questioned by Wood (1969) and more recently by Mauder (1970) who has determined elements for many W Ursae Majoris systems, and found that their components are not in contact. The term "contact system" was first coined by Kuiper (1941), and he used it in a rather different way, namely to signify a close pair of stars surrounded by a common envelope. Lucy (1967b, 1968) has reviveed this usage and has constructed a model for W Ursae Majoris systems in which a common convective envelope plays an essential role. This envelope, although it has to be optically thick, can perhaps be regarded as a special kind of stellar atmosphere.

Before Lucy's work is described in detail, it will be helpful to discuss Eggen's work on the period-colour relation for W Ursae Majoris systems (1961, 1967b). A series of binary systems each of which consists of two main-sequence stars in contact must show a relation between period and colour. Both the masses and radii of main-sequence stars are correlated with their colours, and the sum of their radii determines their separation—if they are in contact. The orbital period is determined by the total mass and the separation, therefore the colours and periods of these systems must be related. A theoretical relation can easily be derived for systems containing spherical stars. Real contact systems contain very distorted stars, and so should obey a rather different relation. Eggen (1961) showed that the effect of distortion is to increase the period corresponding to a given colour. The amount of the increase depends on the degree of central condensation of the component stars, and can be as low as 25 per cent for completely

centrally condensed stars, or as high as 62.5 per cent for homogeneous stars. Eggen was also the first to investigate the empirical relation. In his first paper, he considered as contact systems all those that satisfied the following criteria:

(i) period less than 1.5 days,
(ii) light variation continuous throughout the period,
(iii) light loss at secondary minimum at least 80 per cent of that at primary minimum.

He found that systems with periods less than $0^d.65$ define a period–colour relation, and by applying least squares to his data, I have found that this relation has the approximate form

$$\log P = -0.7(B-V)+0.03 \tag{2}$$

[in an earlier discussion (Batten, 1967) the constant term has been given the wrong sign]. This relation is plotted as a solid line in Fig. 7.1, together with the individual points from which it is determined. The three points corresponding to the longest periods are, in order of decreasing period, μ^1 Scorpii and IM Monocerotis—the only two unreddened systems of early spectral type in Eggen's original list—and H.D. 205372, which was not known at the time of his first paper and for which $(B—V)$ is assumed from the spectral type. Normally, such long-period systems are not regarded as W Ursae Majoris systems which are supposed to be confined to spectral types of late A to G, and H.D. 205372, unlike the W Ursae Majoris systems, contains components of nearly equal mass, but these three do lie near to the period–colour relation defined by the systems of shorter period. Eggen had to apply reddening corrections to all of the other early type systems, and many of them did not lie near this period–colour relation, but on two horizontal branches lying to its left. The suggestion follows that "contact systems" (if indeed they are contact) of early spectral type have a different origin from those of shorter period and later spectral type. At least some long-period systems, however, fit the short-period relation quite well.

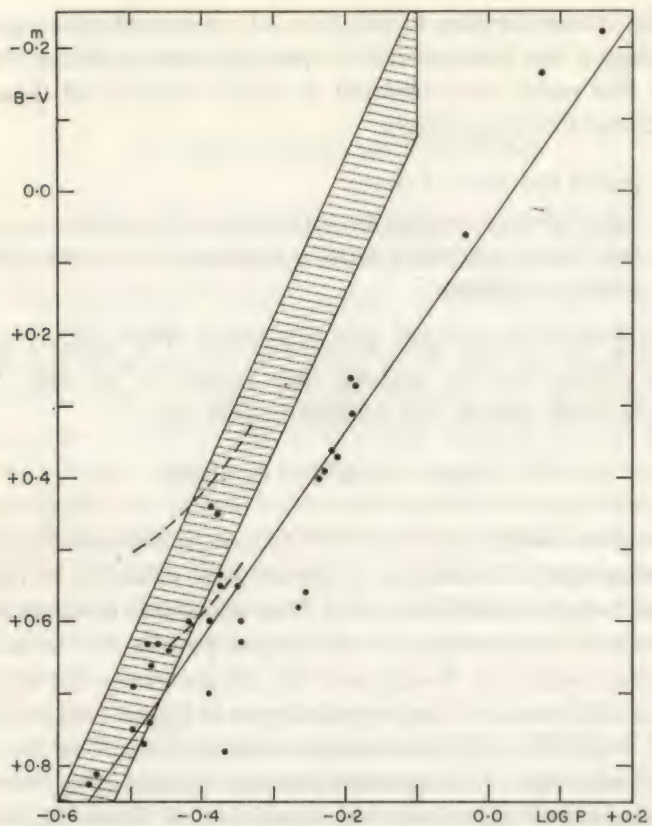

FIG. 7.1. The period–luminosity relation for W Ursae Majoris systems. The points are the uncorrected empirical data, and the line is the mean relation derived from all systems with $P < 1$ day (equation (2) in text). The hatched area is that within which nearly all points lie after corrections made by Eggen and described in the text. The dashed lines represent theoretical relations for two of Lucy's computed sequences of models.

In his second paper, Eggen (1967b) confined his attention to systems with periods less than 1 day and applied two corrections to all the observed colours. The first was a standard reddening correction, usually estimated from nearby field stars, and the second was a correction for line blanketing, to make the observed colours equivalent to

those of the Hyades stars. This latter correction was estimated from the ultraviolet excess of the binaries in accordance with a table published by Wildey *et al.* (1962). The result of applying these corrections is to modify the period–colour relation, and Eggen found that nearly all systems with periods less than a day fall within the shaded band of Fig. 7.1. Within this band, at least among systems of shorter periods, there is evidence for two parallel sequences each fairly close to the respective borders of the band. The left-hand border is a theoretical relation derived for contact systems containing stars whose colours, masses, and undistorted radii are equivalent to those of the Sun, Sirius and VZ Hydrae A, and which are completely centrally condensed. Its equation is

$$\log P = -0.44C + 0.22,$$

where C is the corrected value of $(B-V)$. The majority of W Ursae Majoris systems lie close to this border, and Eggen deduces that their components are, in fact, highly condensed stars. The observed period–colour relation is perhaps the best determined relation for W Ursae Majoris systems, and therefore provides the most critical test of models of these systems.

Kuiper (1941) showed that homogeneous (i.e. unevolved) stars cannot exist in contact unless their masses are equal. The ratio of the radii of the two lobes of the contact equipotential surface is roughly determined by the relation.

$$\frac{R_2}{R_1} \approx \left(\frac{m_2}{m_1}\right)^{0.46},$$

whereas both theory and observation show that for unevolved stars

$$\frac{R_2}{R_1} \approx \left(\frac{m_2}{m_1}\right)^{0.7}.$$

These two relations can only be satisfied simultaneously if $m_2 = m_1$, but it is characteristic of W Ursae Majoris systems that their components have different masses although the luminosities are often nearly equal.

There are three ways to avoid Kuiper's paradox. It may be supposed that at least one of the components is evolved; it may be supposed that the atmospheric structure of the components of W Ursae Majoris systems is different from that of normal stars; or it may be supposed that the components are not in contact. The first of these appears unlikely. Kraft's (1967) estimate that one star in every 2000 is a W Ursae Majoris system, quoted in Chapter 2, indicates that these systems are fairly plentiful. Since their components are rougly similar to lower-main-sequence objects, which evolve slowly, it would be surprising if so many systems were the products of evolution. Moreover, at least one system (TX Cancri) belongs to a galactic cluster, and its position on the Hertzsprung–Russell diagram of that cluster indicates that it is unevolved.

Lucy (1967b, 1968) investigated the second possibility. He envisages a system containing two main-sequence stars in contact, and surrounded by a common, convective, optically thick envelope (Fig. 7.2). Kuiper's argument does not apply to convective envelopes. The adiabatic constant of the envelope differs from that which either star would possess if it were single, and thus the two stars have different equilibrium radii from those of similar single stars. For mathematical purposes, Lucy is able to replace this model by two spherical stars with surface convection zones, and radii slightly in excess of those of the Roche lobes of the contact configuration. If the less massive star derives its energy from the proton–proton reaction, and the more massive star from the carbon–nitrogen cycle, then, provided the convection zones are not too deep, it is possible to construct a contact system containing unevolved stars.

The comparison of Lucy's model with observation provides many points of agreement. For instance, the observed mass-luminosity relation for the components of W Ursae Majoris systems is

$$\frac{L_1}{L_2} = \left(\frac{m_1}{m_2}\right)^\beta$$

where $\beta \approx 1$, whereas $\beta \approx 4$ for normal main-sequence stars. Lucy's model conforms to the observed relation. Although each star releases

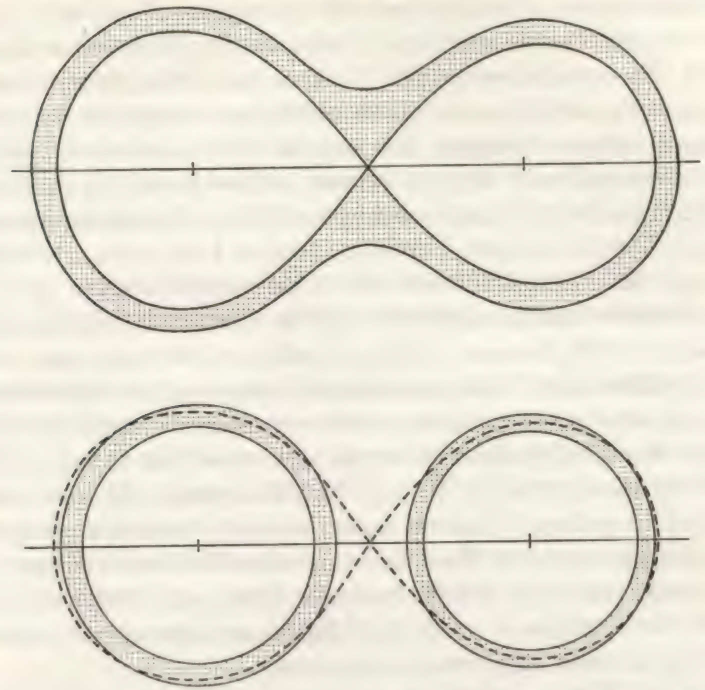

FIG. 7.2. Lucy's model of a W Ursae Majoris system (above) and the mathematically equivalent model containing two spherical stars with which he replaced it. The hatched area represents the convective envelopes—common to both stars in the upper model, and each having the same adiabatic constant in the lower. The Roche lobes have been drawn for a mass ratio of 0.8. The more massive (larger) star also has a central convection zone, where energy is generated by the carbon–nitrogen cycle.

the same amount of energy as it would if it were single, the convective envelope distributes the energy, before emitting it, in accordance with its own equilibrium. Lucy has calculated synthetic light curves for his model (1968) and found that there is a maximum possible depth of eclipse (for contact systems) of about $1^m.3$, and maximum possible difference between eclipses of about $0^m.2$. Both of these are in accord with observations. He finds that the primary eclipse is that of the more massive star, although, according to Binnendijk (1965), this is true

only for W Ursae Majoris systems containing stars of relatively early spectral types. The period–colour relation for Lucy's models is shown by the two dotted lines in Fig. 7.1. Each line refers to a different sequence of models. The two shown are for those sequences of models computed by Lucy that agree best with the observed relation. Neither agrees very well with Eggen's revised relation based on corrected colours, but one does agree quite well with the original uncorrected relation. This is probably fortuitous, because Lucy quite arbitrarily increased the energy-generation rate of the carbon–nitrogen cycle in order to obtain this agreement. Perhaps the most important objection to Lucy's model, however, is the restriction on the total mass of a system implied by it. The less massive star is supposed to release energy by the proton–proton reaction, and cannot, therefore, be much more massive than the Sun. Since mass-ratios very much less than (say) 0.3 are rarely encountered in W Ursae Majoris systems, the total mass cannot be very large. The total masses of Lucy's models range from about $2.2m_\odot$ to $2.6m_\odot$. The observed masses of W Ursae Majoris systems range from about $0.8m_\odot$ to $4m_\odot$. Roxburgh (1966a) tried to explain these systems as products of fission, and was able to predict this observed mass range much more closely.

Moss and Whelan (1970) have attempted to improve Lucy's model by using more recent values of atmospheric opacity. They are still unable to reproduce the observed period–colour relation, and the unevolved system of TX Cancri does not fit any of their unevolved models. Hazlehurst (1970) has investigated models in which the primary is an evolved star. He obtaines better agreement with Eggen's corrected period–colour relation, and can account for a wider range of masses. The system of TX Cancri, however, still represents a problem to this approach. Lucy has also suggested that the systems populating Eggen's secondary period–colour relation (close to the lower boundary of the shaded area of Fig. 7.1) are evolved systems.

The status of contact systems with components of early spectral type is uncertain. The change made by Eggen to the period–colour relation is in the right sense to bring all systems with periods up to $1^d.5$ on to one relation. On the other hand, Eggen points out that the

components of the long-period systems obey the normal mass-luminosity relation ($\beta \approx 4$) rather than that appropriate to W Ursae Majoris systems ($\beta \approx 1$). Lucy argues that these systems must be evolved because any common envelopes they possess will be radiative rather than convective, and the systems are therefore unstable as contact systems. If they are evolved and do lie on the same linear relation as the unevolved W Ursae Majoris systems, this is a puzzling coincidence.

SYSTEMS CONTAINING COMPONENTS SURROUNDED BY EXTENDED ATMOSPHERES

Many binary systems contain at least one component that is surrounded by a very extensive atmosphere. Special methods have been developed to analyse the light curves of eclipsing systems of this kind (Kopal, 1959e; Linnell, 1958, 1961). The spectroscopic study of these systems is also both difficult and rewarding. There are two principal groups of these systems: those containing Wolf–Rayet stars and those containing a supergiant component of late spectral type. In this section they are discussed separately.

WOLF–RAYET SYSTEMS

Although many Wolf–Rayet stars are known or suspected to belong to binary systems (Chapter 2), only a few of these systems have been studied in any detail. The broad emission lines that are characteristic of the spectra of Wolf–Rayet stars strongly suggest that the stars are surrounded by extended atmospheres or shells, and that the matter in these shells is streaming outwards at high velocities (of the order of 1000 km/sec). Underhill estimates that the stars are losing mass at a rate of about $10^{-6}\ m_\odot$/yr (1966). The scant evidence that is available on period changes in these systems does not unambiguously support this figure, although, as explained in Chapter 4, much depends on the effect that the companion may have on the escaping mass.

One of the best studied of these binaries is V 444 Cygni. Kron and Gordon (1950) found that the primary eclipse lasts considerably longer

than the secondary (which is the eclipse of the Wolf–Rayet star). They suggested that the envelope surrounding the Wolf–Rayet star, although not very luminous, is at least partly opaque. Thus it has little effect on the light curve when it is eclipsed, but when the Wolf–Rayet star is in front, the envelope occults the companion and this eclipse lasts longer than the other. Kron and Gordon found that many of their observational results could be explained if the radius of the body of the Wolf–Rayet star is $2.1R\odot$, and it is surrounded by a disk

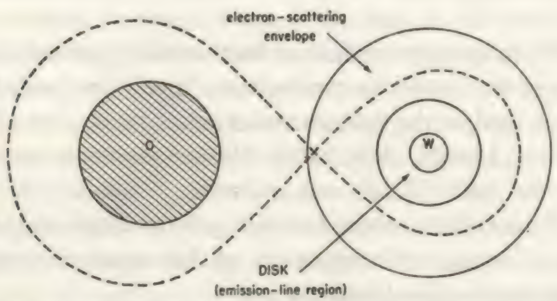

FIG. 7.3. Model of V444 Cygni according to Kron and Gordon (1950). The O-type component is denoted by O, the Wolf–Rayet component by W. The Roche lobes have been drawn in for comparison. An alternative model has most of the matter in the emission-producing disc concentrated in a detached ring. The electron-scattering envelope is probably also detached from the disc.

(or alternatively a detached ring) of radius $7R\odot$, which is in turn surrounded by an envelope consisting mainly of free electrons and of radius $16R\odot$ (Fig. 7.3). Sahade (1958a) pointed out that the proposed envelope extends well beyond the Roche lobe around the star, although the disk does not fill this lobe. He found sharp helium absorption lines in the spectrum which he suggested arose in a more tenuous shell at a still greater distance from the star. Münch (1950) also found supporting spectroscopic evidence for a large envelope surrounding the Wolf–Rayet component.

The idea of such a large envelope is supported by the analysis of a new light curve of V444 Cygni by Cherepashchuk (1966). An interferometric investigation of γ^2 Velorum (Hanbury Brown *et al.*, 1970) also indicates that a large envelope surrounds this Wolf–Rayet star. This investigation was made with the stellar interferometer that was used for the determination of stellar radii (Chapter 5). The system γ^2 Velorum forms a visual binary with γ^1 Velorum. Fortunately, the light of γ^1 Velorum is not sufficient to affect the results, and the whole system can be treated as if it were simply a spectroscopic binary containing a Wolf–Rayet star with a companion of early spectral type. By making observations at λ 4430, the authors have been able to measure the angular diameter of the Wolf–Rayet star. The emission line at λ 4650 (a blend of CIII and CIV) appears to come from a region of larger angular diameter. They were able to measure this angular diameter by making interferometric observations in the light of this line. They found a radius for the star of $17R\odot \pm 3R\odot$, and of $76R\odot \pm 10R\odot$ for the emitting region. This Wolf–Rayet star appears to be much larger than the one in V444 Cygni, its mass also appears to be appreciably larger. Kron and Gordon suggested that the disk that they proposed in their model of V444 Cygni is the region in which the emission lines of the spectrum originate, and this suggestion seems to derive qualitative support from this more recent work. One difficulty in this interpretation is that Hiltner (1950) has found that the strength of the emission line λ 4686 in the spectrum of another system, CQ Cephei, is strongest during eclipses (that is after appropriate corrections have been made for the total light of the system at different phases). This is difficult to reconcile with any simple model in which the source of the emission lines is around either star.

The discrepancy in scale between the radii found for star and disk in the model constructed by Kron and Gordon and from the results of Hanbury Brown *et al.* is not very disturbing in the present state of knowledge. Apart from the fact that the results refer to two different systems, the method of analysis employed by Kron and Gordon is very approximate, while a number of assumptions had to be made (e.g. the light-ratio of the system) to derive any result at all from the interferometric observations. Although the angular measure of the diameter of the star may be quite precise, the measure of its distance,

and therefore the absolute diameter deduced for the star, may be in error. The distance measure depends on the effective temperature assumed, and this is perhaps less certain for Wolf–Rayet stars than for some other objects. There may be considerable differences between the Wolf–Rayet stars in different systems.

The detailed analysis of the light and spectrum variations found in systems containing Wolf–Rayet stars has hardly yet begun. The velocities measured from the lines of different atoms yield different orbital elements for the system. The values of V_0 found for the two stars, or even for the same star from different spectral lines, differ considerably. This may be partly because the emission lines in the spectra are complex blends whose wavelengths are not well known, but it may also be a result of large-scale streaming motions within, or associated with, the disk or envelope. The shape of the velocity curve is sometimes different for the lines of different ions. For example, Hiltner found that the emission-line velocities in the spectrum of H.D. 211863 are satisfied by a circular orbit, but the absorption-line velocities (which are opposite in phase) require an orbital eccentricity of 0.4 (Hiltner, 1945a). This strongly suggests that gaseous streams are present in the system. In the spectrum of CQ Cephei, a similar discrepancy is found between the velocity curves derived from two *emission* lines (Hiltner, 1944): measures of the line of λ 4058 N^{IV} are satisfied by a circular orbit, those of λ 4686 He^{II} require $e = 0.35$. The amplitudes derived from the two lines are also very different. Similar examples are found in the spectra of H.D. 186943 (Hiltner, 1945a) and H.D. 228766 (Hiltner, 1951). Although both the emission and absorption lines in these spectra are difficult to measure accurately, these differences seem to be greater than can be accounted for by errors of measurement, and indicate the presence of streams of large amounts of matter either within the extended atmosphere, or as an extension of it. This conclusion is supported by the light curves of the two Wolf–Rayet systems that have been studied photometrically (V 444 Cygni and CQ Cephei). Both of them show asymmetries in eclipse.

SYSTEMS CONTAINING A SUPERGIANT OF LATE SPECTRAL TYPE

Another important group of systems containing stars with extended atmospheres is that of which the prototype is ζ Aurigae. These systems consist of a supergiant or bright giant of late spectral type, and a much smaller, main-sequence star whose spectral class is usually late B. In addition to ζ Aurigae, the systems 31 and 32 Cygni are well observed, and some fourteen other members of the group are known (Wright, 1970). Most of these other systems are faint, and our knowledge of the group rests almost entirely on the three best-known members. Two other bright stars are sometimes included in the group: VV Cephei and ε Aurigae. Each presents special problems, however, and it is possible that VV Cephei should be more properly regarded as the prototype of another group of systems (Cowley, 1969), while ε Aurigae appears to be unique. Although they are not discussed in great detail in this book, they are for convenience and completeness included in this section.

Two excellent reviews of the ζ Aurigae systems have been published, one by O. C. Wilson (1960) which is chiefly concerned with the structure of the inner atmospheres of these stars, and the other by K. O. Wright (1970)* which is more concerned with the outer reaches of the atmospheres, and the orbital elements of the systems. Much of the material in this section is based on these two articles, which should be consulted for a fuller discussion of these systems. The aim of this section, consistent with the title of the chapter, is to show how the study of these systems has contributed to our knowledge of one particular kind of stellar atmosphere.

The giant component of these systems usually has a radius of some hundreds of times the solar radius (although there is a discrepancy between the precise values deduced from eclipse data and those deduced from the spectral type and absolute luminosity). They are separated from their B-type companions by about 1000 solar radii (several

* Published in 1970, but written some years earlier.

times this for VV Cephei and ε Aurigae). The radii of the companions appear to be normal for their spectral classes, about two to four times the solar radius. The B star is thus essentially a point source of light, and during those phases when its light reaches us through the atmosphere of the giant star, it acts as a probe, revealing to some extent the structure of that atmosphere. It is possible, however, that it is a probe that affects the structure which it reveals.

For some time, it was a matter of controversy whether the partial phases of the eclipses in these systems are atmospheric, or the result of occultation by the opaque body of the giant star. The matter was placed beyond reasonable doubt by the observations of the 1947–8 eclipse of ζ Aurigae made by Roach and Wood (1952). They found that the eclipse began earlier in ultraviolet light than it did in blue light. This is not compatible with occultation by an opaque edge. Still more important, they showed that the difference in behaviour between the light curves in the two colours was quantitatively similar to the observed amount of increased chromospheric absorption in the ultraviolet part of the spectrum. They also found evidence that the giant star had increased in size by about 1 per cent since the previous eclipse.

From spectroscopic observations made during the same eclipse, Wilson and Abt (1954) tried to build a simple model of the atmosphere of the giant star. They studied the changes of ionization with depth, and found that the ionization of iron remains approximately constant, but that of calcium decreases sharply toward the limb. They investigated the effect of the radiation of the companion star on this extended atmosphere, and concluded that the atmosphere must be concentrated in fairly dense agglomerations, otherwise the metals would be much more highly ionized than they are found to be. The ionization behaviour of calcium with depth is anomalous on this theory. Groth (1957) carried out a similar analysis, but he assumed that the influence of the radiation of the companion on ionization in the atmosphere is much less important. If this is so, the state of the atmosphere deduced for giant stars in these systems can be considered typical of giant stars in general.

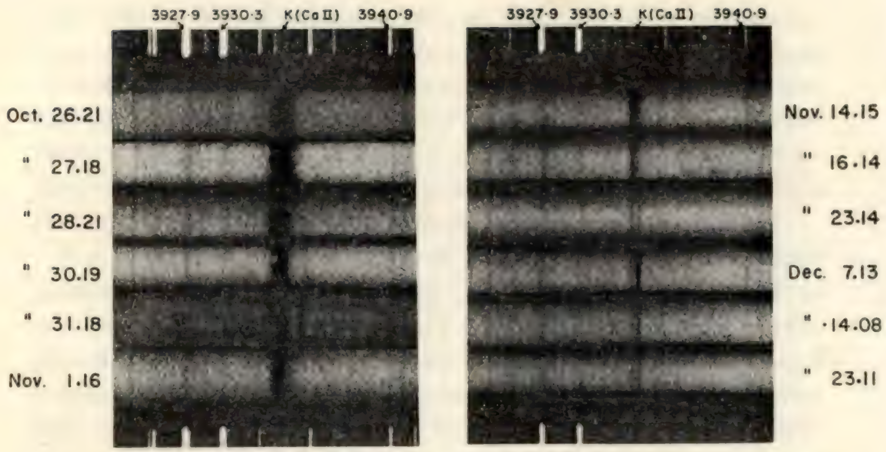

PLATE II. Spectrograms of 31 Cygni showing satellite components of the chromospheric K-line observed in 1951. (Various dispersions: see McKellar *et al.*, 1959.)

The "clumpy" structure deduced for the atmosphere is important, and it has been directly confirmed by observations made about the same time by McKellar and Petrie (1952). When the B star begins to pass behind the atmosphere of the giant star, chromospheric absorptions in the light of many of the metallic lines, and in particular of the K-line of ionized calcium, become visible in the spectrum. Petrie and McKellar discovered satellite absorption lines displaced in either direction from the main component of the K-line by amounts corresponding to velocities of up to 100 km/sec. About the same time, similar satellite absorptions have been found in the spectrum of 31 Cygni (McKellar *et al.*, 1952; McKellar *et al.*, 1959; Wright and Odgers, 1962, see Plate II). At least one such satellite K-line was observed by Wright and Odgers when the projected distance between the centre of the two stars was equal to about five times the radius of the giant star—that is, out to a much greater distance than that to which the atmosphere is normally thought to extend. Wright and Odgers point out that many of these satellite lines have displacements corresponding to velocities greater than that of escape from the giant star.

It is difficult to define the radius of the giant star precisely, partly because the "interface" between star and chromosphere cannot be clearly defined, partly because the star itself may pulsate irregularly, as suggested by the results of Roach and Wood, partly because the contacts of the eclipses are not precisely predictable. The extent of the atmosphere also seems to vary (Welsh 1949), and Wright (1970) has discussed the differences in behaviour of the K-line absorption at different eclipses of the same star, or even between ingress and egress at the same eclipse. There seems to be a tendency for the chromospheric absorptions to be visible for longer before totality than they are afterwards; these extended atmospheres seem to be distinctly asymmetrical. The light curves also vary from one eclipse to another, while that of VV Cephei is disturbed by a semi-regular variation of the giant star which is of the same order of magnitude as the variation caused by the eclipses.

Different values are often derived for the systemic velocities from the spectra of the two components, although this may be partly due to

a systematic error introduced in the spectrophotometric separation of the two components. This effect is also found for VV Cephei from measures of the hydrogen emission lines in that spectrum, which appear to be associated with the secondary component. Peery (1966) remarked that the velocity curve of the primary star appeared to be distorted, as if streams were present in the system, and Wright and Larson (1969) find supporting evidence for this from measures of absorption components of Hα, in the spectrum, that give velocities differing from the expected orbital velocity. The hydrogen emission lines are believed to arise in a disk that surrounds the secondary component, presumably of spectral type B. Thus there is evidence that the extended atmosphere of the giant component is associated with matter streaming between the two stars. The velocity curves of the ζ Aurigae systems may also sometimes be distorted; there is an unexplained discrepancy between the orbital eccentricity found for 31 Cygni by Vinter-Hansen (1944) and Wright (1970). The values are 0.131 ± 0.006 and 0.222 ± 0.008. There may be intermittent streams in these systems, which are related to the clouds found at great distances from the star's surface by Odgers and Wright. Further evidence of clouds is found at secondary eclipse in VV Cephei, when Hynek and Keenan (1945) and Glebocki and Keenan (1967) have found absorption lines of O^{I} in the infrared that do not seem to arise from either of the stars.

Although the giant stars in these systems are very large, the main part of the star itself (the part responsible for the actual eclipse of the secondary) is well inside its Roche lobe. It is not, therefore, easy to understand the apparent instability of these stars. There is, however, considerable evidence for mass loss from late-type giant stars in general (Deutsch, 1969).

There are other systems containing components with extended atmospheres or shells, e.g. V 367 Cygni, ϕ Persei. These are more naturally discussed in the next chapter, although this division is to some extent arbitrary. There is a group of binaries that might well be mentioned here. It consists of systems containing at least one component of G or K spectral type with the H and K lines of ionized calcium in emission in its spectrum. Many single stars of this type are known to show such emission, and it has been used as a luminosity indicator (Wilson and Bappu, 1957). Struve (1946) drew attention to the occurrence of Ca^{II} emission in the spectra of binaries, and Gratton (1950) pointed out that it was found in their spectra regardless of whether the components were giants or dwarfs. Hiltner (1947) published a list of thirteen systems showing this kind of emission, and Popper (1967b, 1970) has published more recent lists. Struve found that the Ca^{II} emission lines in the spectrum of RW Ursae Majoris disappear completely during the annular eclipse of the secondary component. He concluded that they arose in a region at the tip of the tidal bulge, and this idea was developed in Gratton's paper. Vaughan and Zirin (1968) have found that the spectra of many of these systems show strong absorption at the wavelength λ 10,830 He^{I}. It is unclear whether the emission comes from an extended atmosphere, or circumstellar matter, so this group of systems leads naturally into the topic of the next chapter.

CHAPTER 8

OBSERVATIONAL EVIDENCE FOR CIRCUMSTELLAR MATTER IN BINARY SYSTEMS

DEFINITION AND DETECTION OF CIRCUMSTELLAR MATTER

The discussion in the last chapter of systems containing a component surrounded by an extended atmosphere has led naturally to the consideration of streams or clouds of gas in the space between the two components. Many other kinds of binary system show evidence that streams or clouds are associated with them, and it is the purpose of this chapter to summarize that evidence. It is convenient to refer to matter between the component stars as *circumstellar matter*. It is hard to draw any clear distinction between circumstellar matter and extended atmospheres although there are phenomena in some systems that clearly would not be ascribed to any kind of atmosphere. As a useful working definition, circumstellar matter may be defined as matter in the neighbourhood of a binary system that had its origin in one of the components, but is temporarily or permanently no longer under the sole gravitational control of that component.

The existence of circumstellar matter in a given system may be detected in a number of ways. In many systems at least one component fills its Roche lobe, and a strong presumption is thus created that the star is unstable. Circumstellar matter is to be expected in these systems. The changes of period shown by many systems are discussed in Chapter 4, where it is shown that such changes may be evidence either of loss of mass from the whole system, or exchange of mass between the components. Either of these possibilities implies the existence of circumstellar matter. A modern list of systems showing period changes has been compiled by D. B. Wood and Forbes (1963), although many

of the "changes" they list are near the limit of detectability. The existence of circumstellar matter is not only inferred from these observations, however. It can be directly observed if it affects the spectrum of the system by contributing either extra absorption or emission to the total spectrum. Emission can be observed directly. Extra absorption may escape attention unless careful spectrophotometric measurements are made on the spectrum. It may be revealed, however, by radial-velocity measurements since circumstellar components of the absorption lines may distort the whole line profile so that the measured velocity is in error. Distorted velocity curves are therefore evidence for the existence of circumstellar matter. They may be recognized if the system is an eclipsing binary by a difference between the elements e and ω found from the light curve and velocity curve. This effect, already mentioned in Chapter 1, is known as the Barr effect. Distorted velocity curves may also be recognized if two sets of observations obtained at different epochs give different orbital elements: at least one of the curves must be distorted. They may also be recognized if the lines of different atoms or ions in the spectrum of a star give discrepant velocity curves. All these phenomena are evidence of the existence of circumstellar matter. These possibilities have been recognized ever since Struve's pioneer investigations of circumstellar matter. The existence of circumstellar matter can also be inferred from the light curves of many systems. There has been less willingness to accept this possibility, because although a comparatively small amount of matter can produce effects in the limited range of wavelengths of a spectral line, many have doubted whether there is enough matter within the system to produce effects observable in the broad bands of U, B, V photometry. Recently, however, Hansen (1969) has found that electron scattering within a gaseous stream is sufficient to produce effects on the light curve of RZ Scuti comparable to those observed, and similar calculations have been made by Günther for the system SX Cassiopeiae (1959). The matter is discussed further in Chapter 9.

In the remainder of the chapter the available evidence for circumstellar matter is summarized under three heads: the distortion of velocity curves; spectrophotometric evidence, including the existence

of emission features; the distortion of light curves. Within each section, the evidence is summarized for Algol-type systems (that is systems containing a main-sequence primary component of spectral type late B or early A, and a late-type subgiant secondary component); systems containing massive, early type stars (e.g. β Lyrae); the W Ursae Majoris systems; and the cataclysmic variables. Included with the Algol-type systems are the three systems SX and RX Cassiopeiae, and UX Monocerotis. Both components of these systems are giants, although probably neither fills its Roche lobe. The evidence for circumstellar matter in these systems is similar to that found in the Algol-type systems, although the similarity is probably superficial. The evidence that systems containing stars with extended atmospheres also contain circumstellar matter is already presented in Chapter 7.

DISTORTION OF VELOCITY CURVES

ALGOL-TYPE SYSTEMS

Although the velocity curve of Algol itself is not greatly distorted, it was among systems of what is now called the Algol type that distorted velocity curves were first recognized. The best known example is U Cephei (Fig. 8.1). Although the secondary minimum of the light curve of this system is shallow and sometimes distorted, it fairly clearly indicates that $e \cos \omega$ is close to zero. It is usually supposed, although it does not follow rigorously, that e itself is close to zero. Repeatedly, however, velocity curves from quite different epochs have yielded values of $e \cos \omega$ that vary from 0.21 to 0.45, implying values of e in the neighbourhood of 0.2. This sort of distortion of the velocity curve is typical of the Algol-type systems, and also of systems like SX Cassiopeiae. Perhaps an extreme example is S Equulei (Plavec, 1966, 1967b) for which the velocity variation is almost entirely suppressed except just before primary eclipse. The velocity curve of DN Orionis shows some distortion (Smak, 1964a), and since the orbital variation of velocity is small for this system, the distortion looks quite important. The velocity curve of U Sagittae, like that of Algol

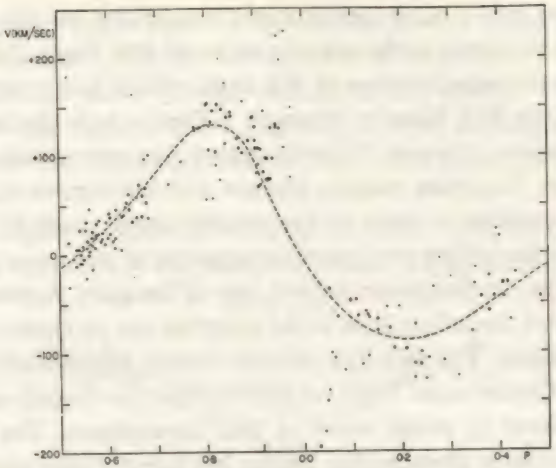

FIG. 8.1. Velocity curve of U Cephei observed at Victoria 1963–8. The dashed line is only a freehand representation of the general trend of velocities. The curve is clearly not of the sinusoidal form that would be expected for a circular orbit.

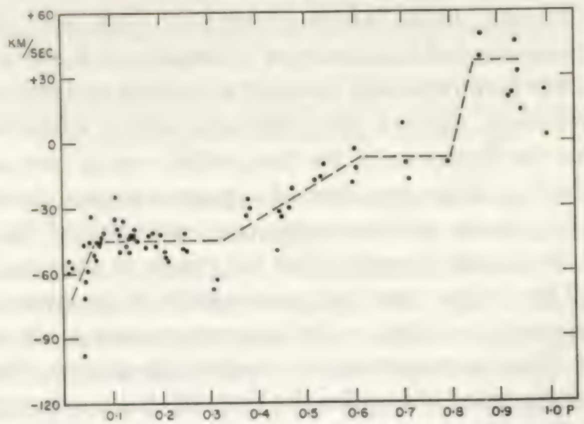

FIG. 8.2. Velocity curve of RZ Scuti from all hydrogen lines except Hα. The step-like appearance is emphasized by the dashed lines. Curve is drawn from data published by Hansen and McNamara (1959).

itself, shows only a small spectroscopic eccentricity, but the distortion is more pronounced in the velocity curve of RW Tauri. The effect is extreme in the velocity curve of RZ Scuti, which has a step-like appearance (Fig. 8.2). Velocity curves of U Cephei look similar but the "steps" are less obvious. Velocity curves like this certainly cannot result from Keplerian motion. Hansen and McNamara suggest that the discontinuities in slope of the velocity curve occur at phases at which some part of the gas stream supposed to be present in the system appears from, or disappears behind, one of the stars. Asymmetries in the profiles of the helium lines in the spectrum are particularly marked at these phases. The step-like velocity curves superficially resemble those of β Cephei stars. Their velocity changes are usually considered to be produced by shock waves in their atmospheres. The existence of shock waves in circumstellar matter has recently been discussed by Biermann (1971), Kříž (1970), and Gorbatskii (1968). In particular, Biermann finds that a shock wave must inevitably develop when a stream of matter meets one of the components, and it is possible that in RZ Scuti we are observing the effect of this.

Struve recognized that these distortions of the velocity curves are observed because the line profiles in the spectra of these systems are distorted at some phases. Hardie (1950) made systematic corrections to Struve's measures of the velocity of U Cephei, and found a plausible circular orbit. Unfortunately, the same correction process, applied to new observations, did not yield the same orbital elements. Struve argued that the distortion of the line profiles was in turn caused by the combination of the spectrum of a gaseous stream, flowing from the secondary to the primary component, with that of the primary star itself. He usually supposed that the stream is seen projected on the disk of the brighter star, and consequently its spectrum is one of absorption which is added to the absorption lines in the spectrum of the star. Thus he tried always to interpret the distorted line profile as containing a broad absorption line from the stellar spectrum, and a sharp line from the stream spectrum. At some phases, however, it seems likely that the stream is seen in emission. Thus, at phase $0^P.75$ (measured from primary eclipse) one would expect to see some of the

stream between the two stars projected on the sky background. The resulting emission probably would not be detectable as an emission line, but would partly fill the absorption-line profile. This profile would also look asymmetrical (Fig. 8.3), but the technique for separating stream and stellar lines would be different from that devised by Hardie.

The examples of distorted velocity curves so far quoted are good individual examples of the Barr effect. Barr (1908) discovered a ten-

FIG. 8.3. Microphotometer tracing of Hα in the spectrum of U Cephei at phase $0^P.79$ (from primary eclipse) superimposed on a similar tracing for phase $0^P.52$ (solid curve). The line looks double at $0^P.79$, but this diagram shows that its profile lies completely within the profile for $0^P.52$. The lower tracing shows the emission that must be present at $0^P.79$ if the difference is caused by emission from the stream between the stars. Red is to the right.

dency for more systems to have values of ω between 0° and 180° than between 180° and 360°. Although Barr's own results were affected by the inclusion of a number of Cepheid variables in his sample, subsequent investigations have confirmed his result. The excess of values of ω is most marked in the first quadrant for Algol-type systems (Fig. 8.4). That this excess is very probably connected with a distortion of the velocity curve is shown by the work of Savedoff (1951) who compared the values of $e \cos \omega$ for several eclipsing systems derived from the light curves and the velocity curves. The light curves indicate nearly circular orbits, but the velocity curves indicate quite large

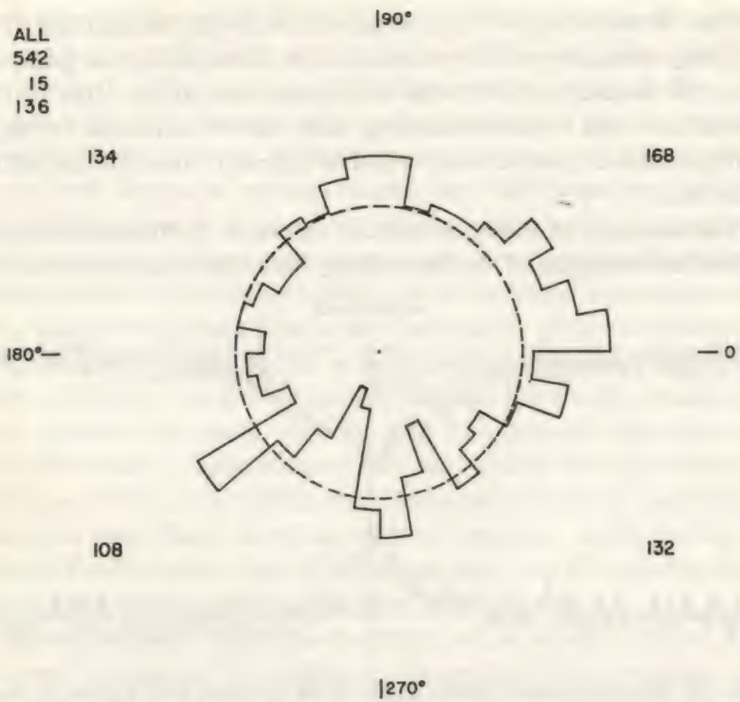

FIG. 8.4. The Barr effect for eclipsing systems with eccentric (spectroscopic) orbits. Because of selection effects this group of systems probably contains a high percentage of Algol-type systems. Presentation of the results is similar to that in Fig. 1.4.

values of $e \cos \omega$. Thus there is considerable statistical evidence that supports the results obtained from systems like U Cephei.

While velocity curves obtained for U Cephei at different epochs do not always agree, thus strengthening the hypothesis that the curve is distorted, the Algol systems in general do not provide many examples of discrepant velocity curves being obtained from different lines in the spectrum of the primary star. This is because the spectrum of the primary component is usually of late B or early A type, and the star itself is often rotating very rapidly, and only the hydrogen lines can be seen and measured. In the spectrum of U Sagittae, however, both Struve (1949) and McNamara (1951b) found differences between the

velocity curves derived from the hydrogen and the metallic lines. The secondary component is virtually dark, so these differences cannot in any way be ascribed to "reflection" of the light of the secondary by the primary; they must rather be considered as further evidence that the hydrogen-line profiles, at least, are distorted at some phases. The K line of ionized calcium in spectra of these systems often has a smaller amplitude of velocity variation than have the hydrogen lines. It is often assumed that this is the result of an interstellar component being blended with a stellar line. This seems an unlikely explanation for stars of late B spectral type, because such stars, at least those in well-observed close binaries, are rarely at a great enough distance for a strong interstellar line to be formed. For instance, U Cephei is only about 400 parsecs away at a galactic latitude of 17°. There are no traces of an interstellar D-line of sodium in its spectrum, but the central component of the K-line—the line has a very complex structure—oscillates in phase with the orbital motion with a total velocity range of 25 to 30 km/sec—probably about one-sixth of the total range of the hydrogen lines, although this is uncertain because the true velocity curve of the system has not been isolated. There is also a small rotation effect (about 70 km/sec) during eclipse. One possible explanation is that the whole system is surrounded by a tenuous cloud, the inner parts travelling with the stars, the outer parts relatively unaffected by the orbital motion. Then the observed K-line velocity would be an average radial velocity along the line of sight through the cloud to the star. If the motions of particles in the portions of the cloud near each star approximate to Keplerian velocity, the observed rotation effect can be used to determine the radius of the *effective* absorbing layer in the cloud. It turns out to be about four times the radius of the star. Reasonable estimates have to be made of the mass and radius of the primary star to reach this figure, but the final result is not very sensitive to them. One argument against this hypothesis is that dilution-sensitive lines should show in the spectrum of such a large cloud. The radius of the hypothetical cloud could be reduced, however, by supposing it to consist of particles spiralling on to the stars, and moving with less than Keplerian velocity.

That the velocity curve of SX Cassiopeiae is distorted in a similar way to that of U Cephei has already been mentioned. The system RX Cassiopeiae contains two stars more nearly equal in luminosity, and orbital elements have been determined from both spectra. They do not agree: the G-type spectrum indicates a circular orbit, while the A-type spectrum yields a velocity curve distorted in the typical Algol fashion. The measured velocities of UX Monocerotis show a large scatter about a curve that can be represented by a circular orbit without doing great violence to the observations. The velocity curve is probably somewhat distorted.

SYSTEMS CONTAINING MASSIVE EARLY-TYPE STARS

In this group of systems, a different kind of distortion of the velocity curve is encountered. For many of these systems both spectra are visible, and the velocity curve of the primary (brighter) component seems to be well determined and undistorted. That of the secondary component, however, does not look like a Keplerian velocity curve at all: the individual observations show a wide scatter about an approximately constant velocity for half the period, and a similar pattern about another constant value for the other half period. A good example is provided by the velocity curve of H.D. 190967 (Fig. 8.5). The constant values are such that the velocity of the secondary component appears to oscillate with the opposite phase from that of the primary component, but the two stars seem to have different values of V_0. Systems like this have been discussed by Sahade (1962); the group includes AO Cassiopeiae, β Scorpii, H.D. 190967 (V 448 Cygni), and H.D. 47129. Since it is clearly impossible for two stars in the same system to have different values of V_0, Sahade supposed that the secondary component (which he regards as the more massive star, even though it is less luminous) fills its Roche lobe and is surrounded by an expanding cloud of gas. Absorption lines in the spectrum of this cloud blend with the same lines in the spectrum of the secondary star so the feature actually measured has a constant shift to the violet

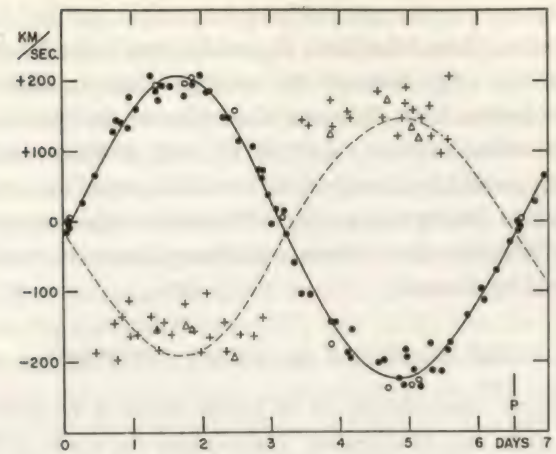

FIG. 8.5. Velocity curve of H.D. 190967 illustrating the tendency of the "secondary component" to have a constant velocity at each node. (Velocity curve was first published by Petrie, 1955.)

relative to the corresponding lines in the spectrum of the primary star. Other massive systems showing this difference in the values of V_0 are H.D. 217312 (Heard and Fernie, 1968) and H.D. 209481 (Petrie, 1962a). Petrie's hypothesis that explains this as a result of errors introduced in the least-squares solution for the orbital elements is described in Chapter 1. The secondary velocity curve of H.D. 209481 is not so obviously distorted as those of Sahade's systems, and the two explanations may be complementary rather than competing. The cloud postulated by Sahade around the secondary components could also account for these stars being the less luminous of their respective pairs, even though they are more massive. Sahade's assumption that they are more massive derives support from the fact that the values he obtains for the masses, using this assumption, are in reasonable accord with theoretical expectations. He also believes that β Lyrae is a member of this group. In this spectrum, emission features from the shell so distort the line profiles of the hydrogen and helium lines, that good velocity curves can only be obtained from the lines of ionized silicon.

Several systems in this group have been observed by Abhyankar (1959) and he has found that both K_1 and K_2 vary in the systems of AO Cassiopeiae. The amplitudes of the hydrogen lines are different from those of the helium lines. Extreme distortion of the velocity curve is shown by the massive binary ϕ Persei (Hynek, 1940), probably again at least partly caused by the effects on measurement of the combination of emission and absorption profiles. This spectacular system has not received the attention that it deserves, although new observations have been reported by Heard.

THE W URSAE MAJORIS SYSTEMS

Spectroscopic observations of W Ursae Majoris systems are very difficult to make and to interpret. The periods are short, and most of the systems are so faint that it is difficult to observe them with adequate time resolution. Thus apparent changes in the values of K_1 and K_2 may only be errors arising from this difficulty. Binnendijk has found such changes in W Ursae Majoris itself (1967). Another spectroscopic problem is that the lines in the spectra usually suffer large rotational broadening. Since most of the stars in these systems are of late spectral types, this rotational broadening is likely to produce a number of unusual blends in these many-lined spectra. If the blends are not recognized and allowed for, they may introduce systematic errors into the velocity measurements and the value determined for V_0 may depend very much on the choice of spectral lines. Lines that give good velocities in normal spectra of the same class cannot necessarily be relied upon in the spectra of W Ursae Majoris systems. This has also been discussed by Binnendijk. In short, although changes in K_1, K_2, and V_0 have been reported for several systems, they cannot be unambiguously ascribed to the effects of circumstellar matter.

CATACLYSMIC VARIABLES

Velocity curves of cataclysmic variables are also difficult to obtain. Often they are based on one or two emission lines, which again may be very broad. The individual observations may show so large a scatter that it is impossible to say whether the velocity curve is distorted or not. One system that does show a difference between the velocity curves of different lines is Nova DQ Herculis. The emission line Hβ shows practically no velocity variation at all. That at Hγ shows a little, while Balmer lines higher than Hγ, and the line λ 4686 He^{II} show a variation in velocity with $K \sim 150$ km/sec. In addition, the He^{II} line shows a rotational disturbance during eclipse (Kraft and Greenstein, 1959). These observations are interpreted by Kraft (1959) as showing that the Hβ emission arises in the nebular ejecta of the nova outburst (as do also the forbidden lines [O^{II}] and [S^{II}]). The Hγ emission, he believes, arises partly there, and partly in a much denser disk surrounding the hotter star, and moving with it in its orbit. The nebular ejecta are supposed to have a high Balmer decrement, and the emission in the light of the higher members of the Balmer series, and the light of λ 4686 is supposed to arise entirely in the dense disk. Since the rotation effect begins and ends with the primary eclipse, it would seem that the eclipse is of the disk, rather than just the star. The disk may contribute a considerable portion of the continuous light of the system by the emission from free–free and free–bound transitions within it.

The system of V Sagittae also shows different velocity curves in different spectral lines. The velocities obtained from measures of the lines of O^{III} differ from those obtained from the lines of O^{IV}, and both differ from the velocities found from the hydrogen lines. There is some evidence that a jet of matter is spiralling out of this system, very like that postulated by Kuiper (1941) for β Lyrae. These results have been obtained by Herbig *et al.* (1965).

SPECTROPHOTOMETRIC EVIDENCE FOR CIRCUMSTELLAR MATTER

Only a rather limited amount of careful spectrophotometric measurement has been made on the spectra of close binary systems, and little progress in the understanding of circumstellar matter can be expected until more results of this kind have been obtained. In the spectra of many systems, however, emission lines have been observed, at least at

some phases, and the detection of such lines may be considered a crude form of spectrophotometry. The discussion in this section, therefore, will include, and even emphasize, the properties of these emissions.

ALGOL-TYPE SYSTEMS

Emission lines in the spectrum of an Algol-type system were first detected by Wyse (1934) and confirmed by Joy (1942, 1947) in the spectrum of RW Tauri. The emission is observed just before and just after totality, and shows primarily in the hydrogen lines, but sometimes also weakly in the K-line and in λ 4481 of Mg^{II}. There is some evidence that the intensity is variable. The emission is not always present in the spectrum of RW Tauri, and is of different apparent intensities when it is observed (Joy, 1947). It has only been observed once in the spectrum of U Sagittae (McNamara, 1951a): in 1969 it could not be detected in this spectrum. Six unsuccessful attempts were made to observe it in the spectrum of U Cephei; a seventh was successful (Batten, 1969). The observations are difficult to make, however, and the detectability of the emission may be very sensitive to both the duration and central time of the exposure. Nevertheless, emission of this sort has been found to be typical of Algol systems. The emission lines seen before total eclipse have a velocity of recession (usually of the order of hundreds of kilometres per second), while those seen afterwards have a velocity of approach of the same order of magnitude. This led Joy (1942) to explain the emission as arising from a detached ring of gas around the primary star. The ring is completely eclipsed by the late-type companion, but is seen briefly in emission at the second and third contacts of eclipse (in some systems the eclipse of the ring is annular). The ring rotates in the same sense as orbital motion, and at second contact the receding portion is seen as red-displaced emission, while at third contact the approaching portion is seen. The ring is visible only during the short time that the light of the primary star has been reduced sufficiently by eclipse, before the ring itself is eclipsed, and during the corresponding interval after eclipse. On this hypothesis, the velocities measured for the emission lines give the velocity of rotation of the ring. Struve found empirically that there is an approximate relation between this observed velocity, V, and the orbital period of the system, P, of the form

$$V^3 \propto 1/P,$$

(although if systems of very long and very short period are included the dependence is different). The size of the ring can also be roughly estimated from the times of appearance and disappearance of the emission. Thus Joy (1942) estimated that the ring in RW Tauri had an outer radius about 1.8 times that of the primary star. Similar, though less well established, values have been obtained for RZ Ophiuchi (Hiltner, 1946) and U Cephei. A different approach was taken to the problem by Plavec (1968b) who tried to reproduce theoretically the observed line profile of the emission in the spectrum of RW Tauri. He assumed that the ring is composed of particles that move in Keplerian orbits about the primary star, and calculated the expected line profiles by an application of Sobolev's theory of moving atmospheres. The emission lines are fairly wide (3.5 A for RW Tauri, and this seems to be a typical value) and the model demands a "ring" with an outer radius 2.6 times that of the primary star. The particle velocities, of course, need not be Keplerian, and the observed Doppler width of the lines could be due to turbulence or convection cells in the ring. Dimensions derived from eclipse timing are probably not very accurate, because of the long exposures required when the emission is visible. Moreover, the essential limitations of the photographic process probably lead to an underestimate of the radius of the ring. The smallest difference in density that can be detected on the photographic plate is determined by the granularity of the emulsion. The granularity is defined as the root-mean-square of the density variation found on a uniformly exposed plate. For a typical emulsion, this can be taken as 0.076.* On the straight-line portion of the characteristic

* This figure and others used in this demonstration are given in a Kodak trade publication, dated 1962, for IIaO emulsion. They are used here only for illustration. It is not intended to imply that they apply either to currently produced IIaO plates, or to any particular plate.

curve, the density D is related to the exposure E by

$$D = \gamma \log (E - E_0),$$

where γ (the contrast) and E_0 are constants for the emulsion if it is developed under standard conditions. A difference in density, ΔD, corresponds to a difference in exposure ΔE given by

$$\Delta D = \gamma \Delta [\log (E - E_0)] = \gamma \Delta E/E.$$

It is reasonable to suppose that for an emission feature to be detected it must cause a change of density on the plate of at least twice the granularity. The value of γ depends partly on the time of development, but $\gamma = 1$ is not a bad approximation. Therefore, the critical difference in exposure needed to detect an emission line is given by

$$\Delta E/E = 0.152.$$

Exposure, in this context, is defined by the product of the intensity of the source, I, and the time of exposure, t. Suppose an exposure to be timed so that the primary star is eclipsed all through the exposure, and the ring is visible all the time. Then, if I_c is the intensity in the continuous spectrum of the secondary star at the wavelength of the emission line, and if I_e is the intensity of the line itself, the emission line receives an exposure

$$E + \Delta E = (I_c + I_e)t,$$

and a neighbouring part of the continuum receives an exposure

$$E = I_c t.$$

Thus the necessary condition for detection of the emission line is

$$I_e/I_c = 0.152,$$

or, the emission line must have approximately 15 per cent of the light of the secondary star (in the same wavelength region) to be detected. If the exposure is not so well timed, and overlaps either with the time when the light of the primary star is making a significant contribution

to the total spectrum, or with the time when the ring is no longer visible, then the ring must be even brighter for the emission line to be detected. If the ring is broad, as Plavec suggests, it is entirely possible that the emission line will not be detectable even though an appreciable portion of the ring has not been eclipsed. Plavec's greater value for the radius of the ring is thus probably an improvement on the older values, because the outer portions of the ring could affect the profile of the emission line when it is visible, even though they cannot produce a detectable emission line of their own. The foregoing demonstration also shows the difficulty of deciding whether the apparent variations of intensity in the emission lines are real or not. In principle, the situation could be improved by using high-contrast emulsions. Unfortunately, such emulsions are usually slow emulsions, and unsuitable for the fairly short exposures that are needed to detect these emissions. A better approach is probably to use scanning techniques and non-photographic detectors, but so far the faintness of these systems, at the important phases, has prevented the application of these new observing methods to this problem.

The inner radius of the ring is even harder to determine. One possible clue exists in the system of U Cephei, because the observed velocity of rotation of the ring is closely similar to that of the primary star itself (Batten, 1969). This suggests that the inner parts of the ring are in contact with the primary star. If this is so, perhaps Joy's model of a comparatively narrow detached ring should be replaced by a broad disk surrounding the primary star, which is immersed in the disk. Struve (1950) has commented that a disk would explain the observations just as well as a ring, and he favoured the idea of a disk for the somewhat different system of UX Monocerotis.

Emission of this kind has never been observed in the spectrum of Algol itself, at least in the normal photographic region. Recently, however, Glushneva and Esipov (1967) have reported that the line of neutral helium at λ 10,830 is visible in emission during eclipse. A red-displaced emission line has been found at Hα at phase $0^P.25$ (measured from primary eclipse—Struve and Sahade, 1957). Sahade (1958b) published a further discussion and showed that the corresponding

feature at phase $0^P.75$ is much weaker. The presence of emission was confirmed by Andrews (1967). It is seen at those phases at which matter between the two stars should be seen projected on the sky background, and therefore in emission. The spectra of some binary systems show doubling of the hydrogen lines at these phases (e.g. RZ Scuti, U Cephei (Fig. 8.3), U Coronae Borealis, in which spectrum the helium lines may be double too) although the secondary spectrum cannot possibly be visible. The interpretation of this in terms of weak emission partly filling the line profile suggested by Batten and Laskarides (1969) is mentioned in the previous section. Struve *et al.* (1957) had made a similar suggestion to explain the observations of U Coronae Borealis. Whether one sees a displaced emission line, or a double absorption line probably depends on the strength of the emission, and the width of the stellar absorption line. At phase $0^P.25$, the stellar line (primary component) is displaced to the violet, and the circumstellar matter, if it is in a stream going from one component to the other, will have little or no radial velocity. In a spectrum like that of Algol, in which the lines are not very broad, the stellar and stream lines are thus seen separately. In a spectrum like that of U Cephei, the displacement of the broad stellar line is insufficient for it to be shifted clear of the stream emission, and an asymmetrical line profile or even an apparently double line, results. Thus the full understanding of distorted velocity curves requires the results of good spectrophotometry. Although the emission observed at quadratures in the spectrum of Algol is only a small fraction of the total light of the system, because it is seen in the full light, it must be quite strong.

From spectrophotometric measures, the equivalent widths of absorption lines in the spectrum of Algol-type systems have been found to be very variable, even at the same phase. The spectral classification can be affected since all lines but those of hydrogen are weak, and may be invisible on some plates. The primary components of these systems seem to have more variable spectra than do single stars of the same spectral range. This may be partly accounted for by the distortion of the components, and by the "reflection effect" (Buerger, 1969), but both effects are expected to be small for these stars.

A spectacular example of the apparent variation of spectral class is provided by RZ Scuti. The primary spectrum has been classified as B3 Ib by Morgan *et al.* (1955) and B0 V by Roman (1956). Kitamura and Sato (1967) find that the equivalent width of Hγ corresponds to a supergiant spectrum. Karetnikov (1967) confirmed the variation—he gave the limits as B3 V and B1 I—and suggested that it depends on orbital phase. Spectrophotometry of the lines provides the clue, although that available until now (Hansen and McNamara, 1959) is in the form of density traces only. The hydrogen lines are fairly broad, but they are partly filled with emission at almost all phases. At some phases only a narrow core of absorption is left, and the spectrum resembles that of a supergiant. If the lines are filled with emission, however, the derivation of an absolute magnitude from the equivalent width of Hγ is in error. The system has been a puzzle for some time. The original light curve by Shapley (1915), when combined with the spectroscopic elements, indicated very low masses for the two components. This light curve was considered poor by Shapley himself, however, and recently Kitamura and Sato have obtained a new (photoelectric) light curve and find that the primary eclipse is a transit, rather than an occultation as supposed by Shapley. The new observations then imply normal masses (i.e. "normal" for main-sequence stars of the same spectral type) but give the primary component the radius of a supergiant. Although there is some spectroscopic support for this conclusion, it is by no means unequivocal, as has just been shown, and the high rotational velocity of the primary star (determined from the rotation effect in eclipse) argues against it.

A further spectrophotometric study of the system of Algol itself has been made by Fletcher (1964) who has measured the equivalent width of the He^{I} line λ 4471 as a function of phase. The measures (which are photoelectric, not photographic) show a large scatter at any given phase. Just before primary eclipse begins, there is a small but definite increase in equivalent width, followed by a sharp decrease during the first part of the eclipse. There is no corresponding increase in equivalent width after eclipse. Fletcher explains these observations by postulating a tongue of matter that extends from the inner surface

of the fainter star, and precedes it a little in orbital motion. This tongue can be identified with part of the stream believed to be flowing toward the primary star. One of the interesting things about this result is that it indicates the effects of circumstellar matter in spectral lines other than those of hydrogen. This is also true of Kitamura's (1967) study of the hydrogen and metallic lines in the spectrum of R Canis Majoris. There appear to be variations in the intensities of these lines even before the eclipse of the primary star begins, and it is likely that this is an effect of circumstellar matter in that system.

The spectra of SX and RX Cassiopeiae and of UX Monocerotis show emission lines of hydrogen that are analogous in many respects to those found in Algol-type spectra (Plate I(c)). They are much stronger, however, and visible even in full light. Indeed, the emission lines in the spectrum of SX Cassiopeiae are stronger in full light, because the emitting source is also partially eclipsed during the eclipse of the primary star. Outside eclipse, this system has an A-type spectrum which resembles that of a supergiant. The dilution-sensitive lines of Mg^{II} and Si^{II} are much weaker than they would be in a supergiant spectrum, however, and Struve has suggested that the star is surrounded by a shell (1950). Detailed quantitative spectroscopy of this system throughout the orbital cycle is needed. Struve has also suggested that in the spectrum of UX Monocerotis the effect of large-scale prominence activity on the G-type component can be observed. He introduced this hypothesis to explain the complex absorption and emission lines of hydrogen and ionized calcium he observed in the spectrum. In particular, it seemed to him to explain the broad, shallow profile of the Ca^{II} lines. Whatever the cause of the phenomena, they are not precisely repetitive, and Struve found large variations in the intensities of the lines of Ca^{II} that are not seen in the lines of hydrogen.

SYSTEMS CONTAINING MASSIVE EARLY-TYPE STARS

All the systems containing massive stars of early spectral type, that were discussed in the previous section, show emission lines in their spectra. Some, such as Plaskett's star (H.D. 47129), show only a weak

emission at Hα. There is, however, other spectrophotometric evidence for circumstellar matter in this system. Galkina (1967 a, b) finds that the equivalent widths of hydrogen lines in its spectrum change throughout the cycle by an amount corresponding to one spectral subclass. Again, this may be either because of a partial filling of the line profile by emission, or because of extra absorption, in either case produced by circumstellar matter. She also finds that the determination of the magnitude difference between the components (by Petrie's method) gives different values when the hydrogen and helium lines are used. This indicates that at least one of the sets of lines has relative intensities different from those of a normal stellar atmosphere, and that therefore they are formed in a region with characteristics different from such an atmosphere. The spectrum of 29 Canis Majoris shows two sets of emission lines (Struve *et al.*, 1958b). One, the emission at Hα, is broad and stationary, although there is some indication of a narrow secondary component. The other set (the "Of emission lines") consists of the lines of N^{III} and He^{II}. These lines are double, but both components are associated with the primary star, and move roughly in phase with it, but with a considerably more positive systemic velocity. Two components of emission are perhaps also to be found in the spectrum of β Lyrae. A broad component may have velocity shifts in phase with the otherwise invisible secondary, while a narrow peaked component is probably associated with streams, or the shell. Recent spectrophotometric measures by Batten and Sahade show that the strength of the broad emission is roughly constant with phase—provided allowance is made for the varying shell absorption and the varying height of the continuum during eclipse. The spectra of the systems V 367 Cygni and W Serpentis show clear evidence that the systems (or at least one star within them) are surrounded by shells.

THE W URSAE MAJORIS SYSTEMS

The spectra of many W Ursae Majoris systems show emission lines of H and K (ionized calcium) in their spectra. These lines are frequently found in emission in the spectra of binary systems of this spectral

class, as is mentioned in the previous chapter. The lines in the spectra of the W Ursae Majoris systems are best seen during eclipse, and Struve suggests (1950) that the source of emission is moving with the more massive component. He also suggested that continuous emission partly fills the lines whenever they are double, because the double components always seem to be weaker than the single lines by an amount greater than expected. Binnendijk (1967) could not confirm this from quantitative measurements in the spectra of VW Cephei and 44 *i* Bootis, but he did find some evidence for the effect in the spectrum of W Ursae Majoris itself. He also failed to confirm unambiguously the statements that the violet-displaced components are always the stronger, and that the spectral types of these systems appear to be later during eclipse (although the systems are undoubtedly redder) especially as judged from λ 4227 of Ca^{I}. Unfortunately, very few good spectrophotometric studies have been made of these systems, and more investigations along the lines of that undertaken by Binnendijk are needed.

CATACLYSMIC VARIABLES

The spectra of nearly all the known binary systems containing cataclysmic variables show emission lines. In the eclipsing systems the hydrogen emission lines usually appear double, and this has often been the way in which further eclipsing binaries have been discovered among these systems. These lines have been interpreted in the same way as those in the spectra of Algol-type systems, and are believed to arise from a disk surrounding the hotter star. In many cases, the secondary component appears to fill its Roche lobe, and the explosive outbursts may provide the means by which more mass is spilled out into the circumstellar region. On the other hand, in some of these systems there is evidence of steady streaming. Because the cataclysmic variables are so much fainter than the Algol-type systems, the emission of the ring, which is not fainter in proportion, can be much more easily detected. The system of V Sagittae appears to be especially complicated. The hot component seems to have expanded to fill its Roche lobe,

and the ring has formed around the cool one (Herbig *et al.*, 1965). This system is anomalous in many ways, however. Its spectrum resembles that of a Wolf–Rayet star, and also shows the double fluorescent emission lines of O^{III} commonly found in the spectra of planetary nebulae. A detailed model has been constructed by Herbig *et al.*: one of its features, a jet of matter spiralling away from the system, is mentioned in the previous section.

EVIDENCE FOR CIRCUMSTELLAR MATTER FROM THE LIGHT CURVES OF ECLIPSING SYSTEMS

Although there are many instances of light curves that show variable or unusual features, it is not immediately clear that these should be attributed to the presence of circumstellar matter. In some systems, a peculiar distribution of light over the surface of one of the components, arising from the distortion of the star and from the "reflection" effect, may account for some unusual feature in the light curve. In others, an intrinsic variation of one or both the stars may be responsible. In the Algol-type systems, the only appreciably distorted component is the secondary, and since this is usually two or three magnitudes fainter than the primary, its effect on the total light of the system is very small. Similarly, the primary component does not receive enough light from it for its own surface-intensity distribution to be affected. If the secondary star varies at all (and there is some evidence that it does in U Cephei) this, again, will not be observable in full light, but only during the total eclipse of the primary star. Thus, unless the primary star varies, peculiarities in the light curves of these systems can be most easily explained in terms of circumstellar matter. It may be that this matter affects the light curve directly, or it may be that if a stream of matter falls on to the primary star, it creates a large bright area, on the surface of that star, that is observable at some phases, and not at others.

ALGOL-TYPE SYSTEMS

Ever since it was first observed, the light curve of U Cephei has shown an unevenness of the two shoulders of primary minimum. The shoulder preceding the eclipse is about $0^m.1$ fainter than that following it (Fig. 8.6). The light of the system diminishes by about that amount in *U*, *B*, and *V* light between phases $0^P.8$ and $0^P.9$. Other less well-

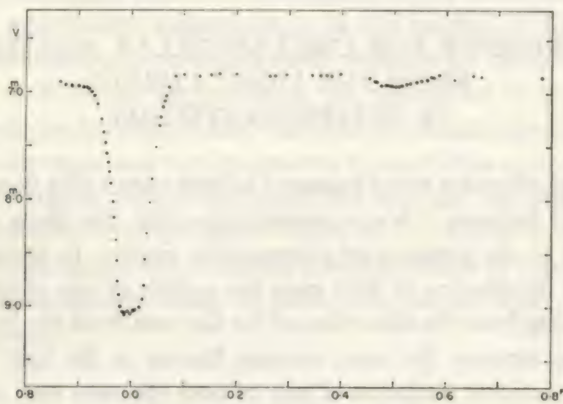

FIG. 8.6. Light curve of U Cephei in *V* light. The plotted points are normal points from Broglia's (unpublished) 1958 observations. Note the difference in the heights of the shoulders of primary minimum and the undisplaced (but distorted) secondary minimum.

established "humps" are found between minima, and the secondary minimum is often distorted. Dugan attempted fairly successfully to explain the principal asymmetry by postulating a tidal bulge on the rapidly rotating tidal star that lagged behind the radius vector joining the two stars. This explanation fell out of favour when Cowling showed (1941) that a normal star can adjust almost instantaneously to external forces, although very recent work by Zahn (1970) shows that tidal lags can occur and may affect the light curve of a system. This asymmetry in the light curve appears in all the observations that have been made, and is remarkably constant. Bolokadze's suggestion (1956) that it is caused by continuous absorption in a stream flowing from the secondary to the primary component appears at least to be plausible.

One of the earliest light curves of RS Canum Venaticorum showed many of the same features (Sitterly, 1930). More recently, the system has been intensively observed at Catania (Fracastoro, 1965; Catalano and Rodonò, 1967) where complete photoelectric light curves have been obtained in four successive years. These curves show a disturbance in the light curve in the form of a depression of about $0^m.1$ to $0^m.2$ amplitude. The remarkable point is that the minimum of the depression seems to occur earlier in phase each year, quite changing the shape of

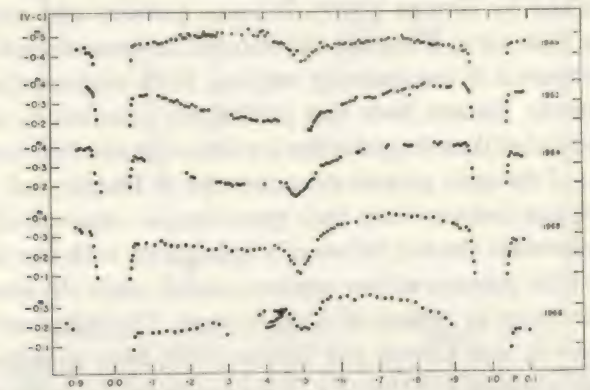

FIG. 8.7. Light curves of RS Canum Venaticorum as plotted by Catalano and Rodonò (1967). The variable distortion is well shown. The 1949 curve was first published by Keller and Limber (1951): the others were obtained at Catania.

the light curve, and particularly of secondary minimum (Fig. 8.7). Catalano and Rodonò suggested that this disturbance is the effect of a ring of gas surrounding the primary star, not in the plane of the orbit, precessing and thus obscuring differing parts of the primary star as seen from the Earth. Further observations have shown that there are difficulties in this hypothesis, but the final explanation is likely to be in terms of some circumstellar structure. This system is not a fully fledged Algol-type system: the secondary component belongs to the group of "undersized" subgiants, and circumstellar matter within the system may not come from it.

Koch (1961) has studied the light curve of TX Ursae Majoris, and found some evidence for its variability. He could not obtain a consistent solution in three colours without assuming the existence of an extended atmosphere around the *secondary* star. Hiltner found some spectroscopic evidence for this (1945b) in the form of a "ζ Aurigae" effect in the K-line during primary eclipse. Asymmetrical hydrogen-line profiles have also been reported in the spectrum (Swensen and McNamara, 1968).

The light curve of RZ Scuti is also peculiar. A brief description has been published by Hansen (1969). Between primary and secondary eclipses the light curve is normal and flat, but between secondary and primary eclipses it is continuously varying. Both minima, therefore, are asymmetric. Hansen finds that preliminary calculations support the hypothesis that these irregularities are caused by electron scattering in a stream of the same general characteristics as Hansen and McNamara (1959) had deduced from their spectroscopic observations. This asymmetry between the two halves of a light curve, such that the first half period after primary eclipse appears normal, while the remainder does not, seems to be typical of Algol systems. The light curve of U Cephei shows it, and Korsch and Walter (1969) have recently noted it in the light curve of AD Herculis. This may be related to the greater prominence found for the Hα emission in the spectrum of Algol at phase $0^P.25$ than at $0^P.75$.

Some anomalies in light curves are variable. The best-known case is probably AR Lacertae, for which Kron (1947) found evidence of spots on the surface of the G5 primary star, presumably similar to sunspots, but much greater. Recently, however, doubt has been cast on this particular case, because Blanco and Catalano (1968) have found that one of the comparison stars that was used is variable, and it is now unclear how much of the variation in the light curve of AR Lacertae is intrinsic to the system. Kron's interpretation required individual spots that cover up to 5 per cent of the visible surface of the star, with as much as 20 per cent being covered at any one time. The largest sunspot yet recorded had an area only just over 1 per cent of the solar disk, and only eight groups having an area of more than

0.3 per cent occur in each solar cycle. A completely dark spot on the sun of area 0.5 per cent of the solar disk would, if viewed from the distance of the nearby stars, be barely detectable as a variation in magnitude of $0^m.005$ (Godoli, 1968). Activity on the scale of solar activity is not likely to be detected on the surface of any star, therefore.

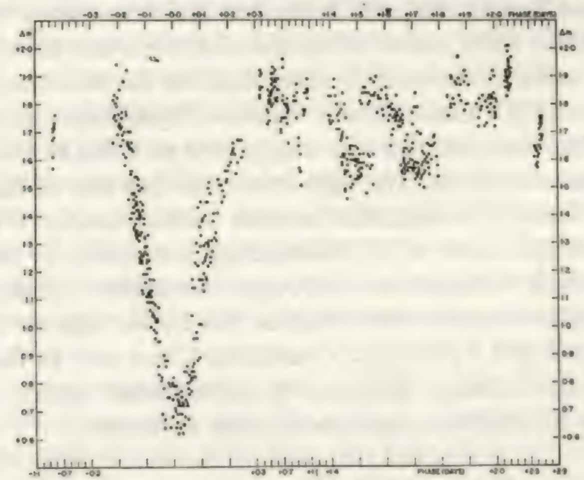

FIG. 8.8. Dugan's light curve of U Coronae Borealis showing a deep secondary minimum never since observed. The time scale is condensed between minima.

Other transient anomalies in light curves are more spectacular. Wood (1957) has discussed the appearance of a hump in the light curve of R Canis Majoris, and Batten (1964) has drawn attention to one light curve of U Coronae Borealis that shows a much deeper secondary eclipse than does any other (Fig. 8.8). Both these features appeared only once. They are both found in visual light curves, but the curves were well observed by competent observers. In both cases, there is some possibility that the appearance of these features was associated with a period change in the system. This would be important evidence that period changes are associated with ejection of a substantial amount of matter, but this sort of observation is difficult to confirm.

The light curve of SX Cassiopeiae is not as well known as perhaps it should be, although the results of an international campaign to be

published soon should help. Old visual light curves show a large ellipticity effect, but the photographic curve does not. This difference is confirmed by a recent three-colour photoelectric light curve (Shao, 1967). Günther (1959) has postulated a detailed model for the system in which the A-type star is supposed to be surrounded by a ring or disk of gas (whose existence is attested spectroscopically from the emission lines) with a radius of four and a half times that of the primary star. His model is discussed in more detail in the next chapter. The light curve of RX Cassiopeiae is variable (Payne-Gaposchkin, 1946) and this has been ascribed to a long-period variation in the light of the A-type component. The light curve also has one of the largest known differences in magnitude between its two maxima (O'Connell, 1951). The light curve of UX Monocerotis is distorted by the effects of an intrinsic variation of the A-type component (Lynds, 1957a). This variation appears to be irregular but Lynds suggests that it is related to the RR Lyrae type of variation. There may be features in the light curve that are produced by circumstellar matter, but it is difficult to sort them out from the intrinsic variation.

SYSTEMS CONTAINING MASSIVE EARLY-TYPE STARS

Many systems in this group are not eclipsing binaries, and they have no light curves from which to infer the existence of circumstellar matter. It might well be of interest to observe such systems as H.D. 47129 and β Scorpii photoelectrically. The former does vary a little, and with good observations it might be possible to study the effects of circumstellar matter freed of the complication of eclipses. The light curve of AO Cassiopeiae is known to vary from one year to another (Wood, 1948), it also shows irregularities and asymmetries (Abhyankar, 1959; Dadaev, 1954). The light curves published by Abhyankar show maxima of unequal height, particularly in ultraviolet light. This phenomenon, well known among eclipsing binaries, is the same as that described for RX Cassiopeiae above. It used to be called the "periastron effect", but Mergentaler (1950) and O'Connell (1951) have shown that it is not in any way related to the orbital eccentricity. Both

Mergentaler and O'Connell suggested that the effect is related to the presence of circumstellar matter in the system, but Milone (1969) —who has coined the term "O'Connell effect"—finds unspecified difficulties in this interpretation.

The light curve of β Lyrae is, of course, well studied. It was recently the subject of an international observing campaign, the results of which have been published by Larsson-Leander (1969). This campaign extended over two observing seasons, 1958 and 1959. The system was found to be both fainter and redder in 1959 than it was in 1958. The extent of asymmetry in the minima changed during this period, and light variations were observed during totality on 1959 August 25 that apparently were not present during the immediately preceding eclipse on August 12. The most cogent photometric evidence for the existence of circumstellar matter in this system, however, comes from polarization measurements. They are discussed more fully in the next chapter.

The light curve of W Serpentis is very peculiar, and displays *three* maxima between consecutive primary minima (Lynds, 1957b). Spectroscopic observations show that this system is surrounded by a well-developed shell. On the other hand, the light curve of V 367 Cygni (Heiser, 1961) shows no such outstanding anomaly, although the spectrum of the system shows that it also is surrounded by a shell. The light curve is asymmetrical; the ascending branch of primary minimum is steeper than the descending branch, and there is some evidence that the depths of minima vary.

THE W URSAE MAJORIS SYSTEMS

Because W Ursae Majoris systems have such short periods they are ideal objects for the photometric observer, and many light curves have been assembled for them from observations obtained in a few nights, or even in one night. Light variations are continuous throughout the period, and the maxima can be of unequal height. The relationship between maxima can change with time as for 44 *i* Bootis (Eggen, 1948). (This is true also of other systems showing the O'Connell effect.) The depths of eclipses can change with time too, as Bookmyer found

for SW Lacertae (1965). She traced this to a change in maximum light, i.e. a change outside eclipse. She found other changes in the light curve that she attributed to continuous absorption and emission arising in circumstellar matter. Binnendijk (1965) has also discussed several cases of changing light curves that could be the result of temporary obscuration of part of the disk of one star. This obscuration could either be a large spot, or circumstellar matter, but the discussion of AR Lacertae makes the spot hypothesis seem unlikely. If there are streams in the W Ursae Majoris systems, it seems less likely that they are contact systems, because if both Roche lobes are full, there should be no streams between them. There have been interesting reports of "flares" (of a few minutes duration) on W Ursae Majoris (Kuhi, 1964), on U Pegasi (Huruhata, 1952), and also shown by Eggen's observations of 44 *i* Bootis.

CATACLYSMIC VARIABLES

Since most cataclysmic variables have only been newly discovered as eclipsing stars, there is not yet a large body of light curves obtained at different epochs from which to obtain evidence of circumstellar matter. Moreover, the components of these systems are known to be variable. The observations are very difficult to make, and the scatter of individual observations is very large. Nova DQ Herculis shows an oscillation in light with a period of 71 seconds. This disappears during the eclipse, and is therefore probably seated in the nova component, since the circumstellar disk whose presence is deduced from emission lines is only partially eclipsed. This system is one of the best observed, and does show small variations in the shape of primary minimum that Kraft (1959) relates to variations in size and shape of the dense disk. The eclipses in many of the systems are very shallow, and can easily be obscured by increased activity of the eruptive component. The light curve of U Geminorum has been followed through several eruptive cycles, and the eclipse is not discernible at maximum light. The light curve of V Sagittae varies from night to night. Other systems show light variations with the spectroscopic period that would

not otherwise be recognized as eclipses (CN Orionis, VV Puppis). In the system of WZ Sagittae, the radial-velocity variation and the light variation are found to be 90° out of phase—primary eclipse occurs at maximum velocity of recession as measured from the hydrogen-emission lines (Krzeminski and Kraft, 1964). The interpretation of these systems must almost certainly require provision for large amounts of circumstellar matter in any models constructed for them.

CHAPTER 9

MODELS FOR CIRCUMSTELLAR MATTER

THE CHARACTERISTIC VOLUME

Many models of circumstellar matter in particular systems have been described and published. They are based on the sort of information that is presented and discussed in the previous chapter. It may be helpful to try to pick out the general trends, and to construct a general model that can be used to describe any system. It is necessary first to have some idea of the amount of space around the binary in which circumstellar matter is to be expected. In a recent review (Batten, 1970) it was proposed to define a *characteristic volume* of a binary system, which was to be taken as a cylinder centred on the centre of mass of the system, having radius equal to twice the major semi-axis of the orbit, and extending above and below the orbit by an amount equal to the radius of the smaller star in the system. The narrow cylinder was chosen because it is commonly supposed that the phenomena of circumstellar matter are best seen in or near the orbital plane, and that therefore the matter itself is confined to that plane. As more evidence accumulates for circumstellar matter in the partially eclipsing system of Algol, and even in the non-eclipsing system of υ Sagittarii, it is less obviously true that the matter is highly concentrated. As is suggested in earlier chapters, the Wolf–Rayet stars, and components of systems like H.D. 47129, are probably losing mass in a spherically, or at least axially, symmetric fashion, with an appreciable portion of the mass being sent out of the orbital plane. Systems like β Lyrae and V 367 Cygni also have well-developed shells that are almost certainly spherical or spheroidal in shape. On the other hand, in the Algol-type systems, matter is almost certainly ejected through the Lagrangian point of the subgiant secondary component. This matter apparently stays close to the orbital plane; even most of the matter ejected above

or below the plane is quickly pulled back (Plavec *et al.*, 1964). There are perhaps two patterns of circumstellar matter. One is a spherically symmetrical distribution that fills a "characteristic sphere" associated with the system; the other is a highly flattened distribution, and the characteristic volume, as already defined, is sufficient for its description.

Within the characteristic volume, three elements can be recognized: *stream*, *disk*, and *cloud* (Fig. 9.1). The *stream* runs from one component

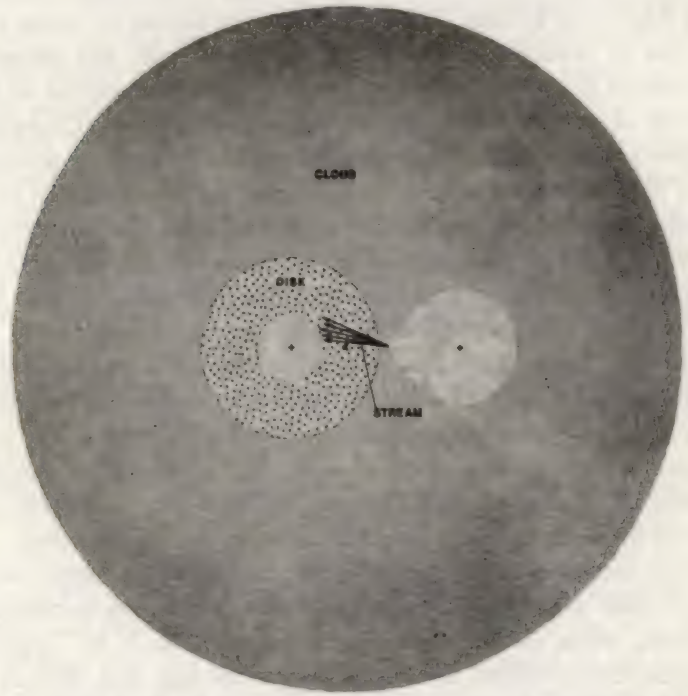

FIG. 9.1. The characteristic volume, and the three elements of circumstellar matter as they may exist in Algol-type systems.

to another, presumably by the shortest possible route in a revolving system, although Hansen and McNamara (1959) have suggested that the stream in RZ Scuti follows a looped course. These streams need

not be continuous either in time or space, but it will be assumed that it is meaningful to speak of their average density. In the Algol-type systems, and possibly also in other systems, major streams will be confined to the orbital plane. The *disk* surrounds one component. In most systems in which a disk is known to be present it surrounds the more massive component. There is no reason in principle, however, why both components should not have disks. The disks are probably fed by streams; experience shows that they, too, are confined to the orbital plane. The disks can be identified with the emitting structures in the Algol-type systems. These structures are known to rotate with velocities of several hundred kilometres per second, and therefore are expected to be flattened. Many authors have commented on the similarity of these structures to those surrounding Be stars. Coyne and Kruszewski (1969) have shown that the measured degree of polarization of light from a Be star can be accounted for by a disk that has a thickness only a small fraction of a stellar radius. In the previous chapter, evidence is presented that the disk may be in contact with the central star, and extend to three or four times the stellar radius. The *cloud* is supposed to surround the whole system. It is probably the same as what is called the *shell* in many systems. It is this element of the circumstellar matter that is most likely to extend out of the orbital plane, and perhaps to approximate to spherical symmetry. The task of constructing a model of the circumstellar matter is to estimate the densities of each of these elements, to trace out, if possible, the course of the streams, and to estimate the outer radius of the disc.

It is not necessary that all these elements are present in every system. Not everyone will agree that the Algol-type systems are surrounded by a cloud, while, on the other hand, in a system like V 367 Cygni in which the shell spectrum dominates that of the star, it is difficult to know what may lie between the shell and the star. Rather, these elements should be sufficient to provide a full description of all systems. A model that supposes that disk and cloud each have their own uniform density, with a sharp boundary between them, is obviously artificial. A similar model seems helpful in understanding the spectra of Be stars, however (Hutchings, 1970), and the crude model may not

be a bad first approximation. The true distinction between disk and cloud is kinematical. The stars will move *with* the disk, but *through* the cloud. Particles in the cloud (ignoring collisions) will move under the influence of both components: those in the disk will be under the influence of only the one component within.

The streams are likely to be the densest of the three elements: they arise directly from the atmosphere of one of the stars. Korsch and Walter (1969) have pointed out, however, that the density of a stream may vary appreciably along its length, because as matter in the stream travels from one component to the other it accelerates, and the end of the stream is less dense than the beginning. The matter in the disk will, presumably, eventually descend on the component it surrounds. Indeed, one function of the disk may be to store matter and angular momentum until the receiving component can accommodate it. The relative density of stream and disk will depend on the rates at which matter is fed into the disk from the stream and at which it falls on to the star. The cloud, at least in systems in which the circumstellar matter comes mostly through the Lagrangian point, will be the least dense of all. It will come from matter that "diffuses" out from the disk and streams, and from any matter that may escape from other points on the stars' surfaces. The relative visibility of streams, disk, and cloud will depend on many factors—the spectral types of both stars, their separation, the different states of ionization and excitation in the different elements of the circumstellar matter itself. One factor, however, will be the relative masses of the elements. The mass of a stream will be roughly proportional to its length, and therefore to the separation between the two stars. The volume of the disk and (in Algol-type systems) the cloud will be proportional to the square of the separation of the two stars, if these two elements are confined to the orbital plane. From Kepler's third law, then,

$$m_{\text{stream}} \propto P^{2/3},$$

$$m_{\text{cloud and disk}} \propto P^{4/3}.$$

Clouds and disks, therefore, are likely to be relatively more prominent in long-period systems: streams will be more prominent in short-period

systems. This expectation is fulfilled, to a considerable extent. The Algol-type systems, in which streams materially affect the spectrum even in full light, have periods of only a few days. The existence of a cloud is conjectural, and the disk can usually be seen only with difficulty during the primary eclipse. In the very short-period U Geminorum systems, streams play a very important part in the spectrum, although the disks are more easily seen than in the Algol-type systems, possibly because of the lower luminosity of the component stars. In the spectrum of β Lyrae (orbital period about 13 days), streams, disk, and shell can all be recognized. The spectrum of V 367 Cygni (orbital period 18.6 days) is dominated by the shell. On the other hand, although the spectra of SX and RX Cassiopeiae show shell features, and the disk is quite prominent, the stream effects are still appreciable. The periods of these systems are 36 and 32 days respectively. The situation is similar in the spectrum of VV Cephei (period 7450 days: Wright and Larson, 1968). The correlation is not perfect: some of the other factors mentioned are important, and the dependence of the relative visibility of streams, disks, and clouds on their masses is not necessarily linear.

THE DENSITY OF CIRCUMSTELLAR MATTER

Several estimates of the density of circumstellar matter in different systems have been made by a variety of methods. Until recently these were scattered through the literature, until many of them were gathered in a recent review article (Batten, 1970). The table presented in that article is repeated here (Table 12), because a few more estimates that were overlooked then have been added, and at least one value has been modified. Some of the newly included estimates are particularly valuable because they have been reached by methods different from those used for the others, but they tend to confirm the general conclusions. Some of the estimates in Table 12 are my own deductions from the published data. The original authors may not wish to take the step of converting their data (in all cases the number of Balmer emission lines visible) into electron densities. These deductions are clearly

TABLE 12. ESTIMATES OF CIRCUMSTELLAR DENSITY IN CLOSE BINARY SYSTEMS

System	Stream, cloud or disk	Density of	Value (cm^{-3})	Method	Reference
typical	average	particles	10^{16}	estimate	Struve (1951)
AE Aqr	disk	particles	6×10^{10}	mass-transfer rate	Crawford and Kraft (1956)
ε Aur	cloud	electrons	10^{11}	Inglis–Teller	based on Hack (1958)
SX Cas	disk	electrons	1.6×10^{12}	light curve	Günther (1959)
U Cep	stream	particles	2.5×10^{14}	see text	
	disk	electrons	1.7×10^{14}	Inglis–Teller	Batten (1969)
VV Cep	cloud	electrons	6×10^{11}	Inglis–Teller	Wright (unpublished)
	disk	particles	1.2×10^{14}	model	Hutchings (unpublished)
	cloud	particles	1.2×10^{11}	calculations	
U Cr B	stream	electrons	8×10^{12}	Inglis–Teller Hβ emission	based on Struve *et al.* (1957)
Nova T Cr B	disk	electrons	2×10^{10}	Inglis–Teller	Kraft (1958)
V367 Cyg	cloud	electrons	6×10^{12}		Heiser (1961)
AD Her	stream	particles	10^{12}–10^{14}	effect on light curve	Korsch and Walter (1969)
		electrons	$\approx10^{12}$		
Nova DQ Her	disk	electrons	3×10^{13}	emission near Hβ	Kraft (1959)
β Lyr	disk	electrons	4.4×10^{10}	polarization	Shulov (1967)
	stream?	electrons	$\approx10^{11}$	light curve	Dadaev (1954)
WZ Sge	stream and disk	electrons	4.5×10^{12}	Balmer emission intensity	Krzeminski and Kraft (1964)
	stream	particles	$\approx10^{11}$	k.e. of stream	Gorbatskii 1967
υ Sgr	stream	particles	3×10^{9}	condition for laminar flow	Nariai (1967)
RZ Sct	stream	particles	10^{12}	estimate	Hansen and McNamara (1959)

TABLE 12 *(cont.)*

System	Stream, cloud or disk	Density of	Value (cm^{-3})	Method	Reference
RZ Sct	stream?	electrons	10^{14}	Inglis–Teller	Karetnikov (1967)
	stream	atoms	10^{11}	equiv. width absorption lines	Hansen and McNamara (1960)
W Ser	cloud	electrons	4×10^{11}	Inglis–Teller	based on Hack (1958)
RW Tau	disk	electrons	2.4×10^{13}	Inglis–Teller	Plavec (1968a)
H.D. 47129	cloud	particles	3.2×10^{11}	curve of growth	Abhyankar (1959)

indicated in the final column of the table where they are describing as "Based on . . .". The agreement between all the estimates is encouraging, only eight of them lie outside the range 10^{11} to 10^{14} particles per cubic centimetre (if the distinction between particle density and electron density is ignored for a while). The term "particles" refers to ions, atoms, and electrons. Of these eight estimates, that by Struve was intended only as an order of magnitude estimate, and is included in the table only for completeness. He roughly estimated the total mass and total volume of clouds in a typical binary system. The lowest estimate in the table, that for υ Sagittarii, is not empirical. It is the condition for laminar flow through the Lagrangian point. The assumption of laminar flow makes a discussion of the system theoretically tractable, but there is little or no evidence for it. The other six estimates lie only just outside the quoted range: they are discussed in the following paragraph.

The method of estimating circumstellar density that has been most frequently used is the application of the Inglis–Teller formula relating the last line of the Balmer series that is separately visible to the electron density. It has been applied to the emission lines seen in the spectrum

of cloud, stream, or disk. (It is not always possible to be sure from the published work to which of these elements of circumstellar matter a given estimate applies. Different authors have also used slightly different values for the constant term in the Inglis–Teller formula, so not all these estimates are completely uniform.) The derivation of the Inglis–Teller formula depends on the assumption of local thermodynamic equilibrium, which is certainly not fulfilled in circumstellar matter. The results are offered here, not as final well-established figures, but rather as preliminary values that will eventually enable a more realistic model of the circumstellar matter to be built. It is at least encouraging that agreement between figures derived from the Inglis–Teller formula and those derived by other means (not all of which include any assumption about thermodynamic equilibrium) is as good as it is. Another possible source of error in the application of the Inglis–Teller formula is the breadth of the emission lines (in Algol-type systems they are about 3 Å wide). Broad lines increase the probability that the number of resolvable lines will be underestimated, and the electron density, therefore, will be overestimated. Again, the results obtained by other methods are reassuring that no great error has been committed in this way. This effect has been estimated quantitatively by Kurochka (1968) who concludes that for electron densities greater than 10^{11} cm^{-3}, the correction factor is less than 3.

The estimate by Günther for the disk in SX Cassiopeiae is made directly from the light curve. The secondary minimum (when the large G-type star is eclipsed) is much broader than the primary minimum, and Günther explains this by postulating a disk around the component of spectral type A, that is dense enough to diminish the light of the secondary star even when no part of it is eclipsed by the primary star itself. The disk is not so luminous (at least in the continuous spectrum) that its own eclipse modifies the shape of the light curve. There is independent evidence for the existence of such a disk in the emission lines of hydrogen observed in the spectrum of this system. From the durations of the two eclipses, Günther finds that the radius of the disk must be about 4.65 times the radius of the A-type star. If the

opacity of the disk is caused by electron scattering, then he finds the value of the electron density given in Table 12. Günther also attempts to calculate the density of the disk from the observed intensities of the emission lines. He finds a lower limit to the total number of hydrogen atoms of 1.1×10^9 per cubic centimetre, which implies a degree of ionization (N_e/N_H) of 1.4×10^3. If Saha's theory is used to predict the degree of ionization, due allowance being made for the dilution of radiation, it is found to be much lower. The same discrepancy can be expressed in another way: the degree of ionization of the disc calculated by the first method corresponds to an ionization temperature of 11,500°K, while the effective temperature of the A6 star is only 8400°K. There are two possible sources of this discrepancy: there is as yet no precise spectrophotometry of the emission lines in the spectrum of SX Cassiopeiae, and the applicability of Saha's theory to the disk is highly questionable.

The unpublished estimate by Hutchings for the disk of VV Cephei has been obtained by matching the observed profile of the emission lines with profiles computed for various assumed values of the size and density of the disk. He finds that the density decreases quite rapidly outwards. The density at $100R_\odot$ from the Be component is 0.1 per cent of that near the surface of the star. This lower density is quoted in the table as a "cloud" density, although there is probably no clear distinction between cloud and disk. The agreement of this cloud density with that derived by Wright from the Inglis–Teller formula is very satisfactory. The disk density only just exceeds 10^{14} particles/cm^3.

The estimate of 2.5×10^{14} particles/cm^3 in the stream of U Cephei is one of the highest in the table. It is discussed in more detail in a later section of this chapter. It agrees well with the independent result obtained by applying the Inglis–Teller formula to the emission lines observed at eclipse (Batten, 1969).

The density of the disk of Nova T Coronae Borealis as obtained by Kraft (1958) is one of the lower figures in Table 12. He assumes that all transitions producing Hβ are the result of electrons that have just been captured and then descend from the upper level of the transition. He supposes that the emission comes from a spherical volume surrounding the hot star (the estimate of density would be increased several times if the emission comes from a cylindrical volume with a thickness 0.1 times its radius). Probably, however, not all the electrons in the upper level of the Hβ transition arrived there because they had just been caught, and the electron density is overestimated if it is assumed they have been. Kraft calculates that the luminosity of the blue star could be maintained by accretion from a disk of density of 7.4×10^{10} electrons/cm^3. The true electron density in the disk is thus probably somewhere between the value given in Table 12 and this latter value, and it does seem likely that the disk in Nova T Corona Borealis is somewhat less dense than those in other systems. This result would not be surprising by itself, because there is no reason why circumstellar matter in ex-novae should follow the same pattern as in Algol-type systems. It is surprising, however, when compared with systems like Nova DQ Herculis and WZ Sagittae, which although similar, appear to be surrounded by denser matter. The electron density in the former system is determined on the assumption that emission detected near Hβ arises from the Paschen continuum (and checked by a computation based on the emission intensity of λ 4686 He^{II} similar in principle to the method just described for Hβ in the spectrum of T Coronae Borealis): that in the latter is estimated in essentially the same way as for T Coronae Borealis, except that Hγ was used instead of Hβ. (The other estimate for WZ Sagittae, by Gorbatskii, is based on the assumption that most of the observed luminosity is derived from the kinetic energy of the stream.) On the other hand, the density estimated for the system AE Aquarii is similarly low. This depends on the estimated rate of mass transfer in the system ($\sim 10^{25}$ g/yr, which leads to a density of the order of 10^{-13} g/cm^3). There is also evidence in the spectrum of T Coronae Borealis from the forbidden lines of [Ne^{III}] for a cloud surrounding the system. Kraft believes that it must be located beyond the outer Lagrangian surface of the system, that it has a density of the order of 10^7 electrons/cm^3, and that it consists of matter escaping from the system. As already mentioned in Chapter 8, the matter is probably spiralling out from the system in a way similar to that proposed by Kuiper (1941) for β Lyrae.

The stream density in the system AD Herculis is estimated from the effect of the stream on the light curve at phase 200° on the assumption that the opacity of the stream is caused by electron scattering. The ionization in the stream has been estimated at various points along the stream. Korsch and Walter believe that the ionization increases near the hot star, and although they point out that the particle density must be lower in the more rapidly moving parts of the stream, and can therefore vary along the stream by a factor of 100, they find a nearly constant electron density along the stream of the order of 10^{12} electrons/cm^3.

The result for the disk around the apparent secondary of β Lyrae is obtained from measures of polarization. Several investigators have recently published polarization studies of this system (Appenzeller, 1965; Ruciński, 1966; Coyne, 1970; Shulov, 1967). All agree that the observed variation in polarization is in agreement with the hypothesis advanced by Huang (1963) that there is a disk surrounding the less luminous, but probably more massive star of the system. This hypothesis was put forward to explain the fact that the light curve of the system resembles that of a contact system, but the relative dimensions derived for the system are not those of a contact configuration. Woolf (1965) suggested that if this disk contained about half the mass of the invisible component (about $10m_\odot$) the underluminosity of this star could be explained. Only Shulov has attempted to derive the density and mass of the disk from the polarization observations. He finds a total mass of the disk of $10^{-7}m_\odot$, very much less than is required by Woolf's version of Huang's hypothesis. To derive the electron density, he has to assume a model of the disk, and a value for its volume. For that reason, the electron density given in Table 12 may be uncertain by an appreciable factor. His results may also be affected by uncertainty in the interstellar polarization, which appears to be similar both in direction and total polarization to that found in the light of the system itself. It seems unlikely, however, that uncertainties of this kind can account for the factor of 10^8 in the mass of the disk required by Woolf's hypothesis. Dadaev's (1954) estimate of the stream density in this system was made by a method similar to that used by Günther for the disk of SX Cassiopeiae.

Different methods of estimating the electron density in the system of RZ Scuti have produced different results. Karetnikov (1967) used the Inglis–Teller formula and found that the density of the stream varied with phase by a factor of 5 (the mean value is shown in Table 12). If different parts of the stream are seen at different phases, this observation could be regarded as confirmation of the hypothesis put forward by Korsch and Walter that the density of the stream varies with its velocity. A rough estimate by Hansen and McNamara (1959) is rather lower than any of Karetnikov's values, and a more careful estimate by the same authors (1960) leads to a lower value too. This last estimate is based on the analysis of a component to the hydrogen line in the spectrum that is believed to arise in the stream. The number of hydrogen atoms producing the line is calculated, and if reasonable assumptions are made about the dimensions of the stream, its density can be found. Further investigation of this spectrum is very desirable.

The particle density given for the cloud around H.D. 47129 is also obtained from spectrophotometric measures of absorption lines. Abhyankar has made a curve-of-growth analysis of the shell spectrum, and found the number of hydrogen atoms, helium atoms, and electrons present in the shell. All these are added together to give the particle density. He finds that almost all of the hydrogen and helium are ionized, and most of the helium (90 per cent) is doubly ionized. The application of the curve of growth to the shell is, of course, open to the same objections about local thermodynamic equilibrium as is the application of the Inglis–Teller formula, and it can only be justified as a first approximation used in order to obtain some idea of how far conditions may depart from equilibrium.

In Table 12 estimates of the electron density and particle density are entered together. The relation between these two quantities is determined by the degree of ionization of the circumstellar matter, and it is important to know this if the density of the circumstellar matter is to be fully determined. Some empirical estimates of the degree of ionization have been made. Abhyankar's conclusion that both hydrogen and helium in the shell of H.D. 47129 are virtually completely ionized has already been mentioned. Krzeminski and Kraft (1964) found that the

stream in WZ Sagittae is also virtually completely ionized, but this very short-period binary is certainly a special case. Günther found nearly complete ionization in the disk of SX Cassiopeiae, but it was just on this question of ionization that his model was found to be inadequate. Dadaev's results for β Lyrae also imply a high degree of ionization. Crawford and Kraft assumed that the streams in AE Aquarii are completely ionized because there is no self-absorption in the emission lines. If all these estimates are correct, particle densities and electron densities are quite comparable and can be used interchangeably. The circumstellar matter, like the stars it comes from, must be composed predominantly of hydrogen and helium, and therefore the particle density is approximately twice the electron density, if the matter is almost completely ionized. The estimates in Table 12 are not accurate to within a factor of 2. The mean density of each element of the circumstellar matter can therefore be estimated by taking simple arithmetic means from Table 12. Streams and disks both have densities of the order 10^{13} particles/cm^3, and clouds of the order of 10^{11} particles/cm^3 or less (except in V 367 Cygni which has a much denser cloud). These figures are probably uncertain by at least one power of 10, and there probably is a real variation from one system to another. Even within a given stream, there may be quite a large variation of density, as Korsch and Walter have pointed out. The densities in the disks of Be stars are probably about 10^{14} particles/cm^3 in their densest portions (Hutchings, 1970). Underhill has estimated 10^{12} particles/cm^3 as a plausible density for the shells around Wolf–Rayet stars (1966). The density of 10^{13} particles/cm^3 is about 10^{-3} of the density of the reversing layer of a solar-type star (Allen, 1963). The densities deduced for circumstellar matter in binary systems are thus reasonable in comparison with those found for other, similar structures.

EFFECTS OF STREAMS ON LIGHT CURVES

It has for some time been a question of dispute whether or not the streams postulated to explain the spectroscopic observations of close binary systems can be expected to affect the light curves of these sys-

tems. The argument has been that comparatively little matter is needed to produce an observable effect in the light of one spectrum line, but much more would be needed to produce continuous absorption or emission that could be detected in broad-band U, B, V photometry. Since calculations have been published some time ago (Dadaev, 1954; Günther, 1959) that show it is entirely reasonable to expect effects on the light curve from streams and disks, it is a rather sad comment on the efficiency of scientific communication that the question should still be disputed.

If the result of the previous section is accepted, and the matter in streams is considered to be almost completely ionized, then the dominant cause of opacity in the streams must be Thomson scattering by free electrons. The coefficient σ_{el} for this form of scattering is given by a standard formula (see for example, Unsöld, 1955)

$$\begin{aligned}\sigma_{el} &= \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 N \\ &= 0.665\times 10^{-24} N,\end{aligned}$$

where e and m are the charge and mass of the electron, respectively, c is the velocity of light, and N is the total number of electrons encountered by the incident ray. The ratio of the emergent flux, I, to the incident flux, I_0, is then given by $I/I_0 = \exp(-\sigma_{el})$.

Consider a plane parallel stream of thickness one solar radius (7×10^{10} cm) and electron density 10^{13}/cm^3 that completely occults a star. Light from each square centimetre of the star's surface is then intercepted by 7×10^{23} electrons, and $\sigma_{el} = 0.465$. The ratio (I/I_0) is then 0.628, and the diminution of the total light of the star is $0^m.51$. This is considerably greater than irregularities of the light curves observed in many systems. In Chapter 8 a drop in magnitude of $0^m.1$ in the light of U Cephei is ascribed to obscuration by a stream that seems to be denser than 10^{13} particles/cm^3. If the light beam were intercepted by only one-tenth of the above number of electrons, *either* because the stream is thinner, *or* less dense, *or* because it obscures only part of the stellar disk, then the effect of scattering would be to

make the star appear only $0^m.05$ fainter. Although streams in binary systems are not plane parallel layers, it is quite possible that, at some phases, all or part of one of the stars is obscured by a stream whose thickness is comparable to the solar radius. Thus both the dimensions and densities of observed streams seem to be of the right order to produce easily detectable effects in the light curves of eclipsing binaries.

THE STANDARD STREAM

In this section the relation between the density of the stream and the amount of matter transferred between the two components is explored. To do this, some assumptions about the size of the stream and the speed with which matter passes through it are needed. Con-

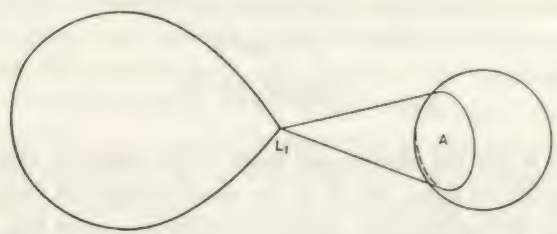

FIG. 9.2. The standard stream. The area A is not necessarily circular and may be partly on the hidden side of the star (the drawing is for phase $0^P.75$), but it is equal to the area of the projected solar disk.

sider a stream that diverges from the Lagrangian point of the star losing mass (Fig. 9.2). Let the cross-sectional area of the stream, where it impinges on the mass-gaining star be A, equal to the area of a disk having as radius one solar radius. Let the velocity of matter as it leaves the stream and falls on the mass-gaining star be V_f. From observation it is known that it is convenient to express V_f in units of 100 km/sec. Let it further be assumed that the stream is not used for the storage of material, that is that the amount of matter leaving the stream is at all times equal to the amount of matter entering it. If the

stream contains n particles/unit volume, then the total mass leaving the stream in unit time is

$$nAV_f$$

and equilibrium is reached when

$$nAV_f = N,$$

where N is the number of particles lost in unit time from the mass-losing star. Or, under the assumptions adopted, the particle density of the stream is given by

$$n = \frac{N}{AV_f}.$$

The length of the stream, and the initial velocity of the particles, do not enter explicitly into this equation, although they do enter implicitly through the quantity V_f. The quantity V_f is more likely to be known observationally than either the length of the stream, or the velocity of ejection from the mass-losing star. In fact, V_f is not very sensitive to either of these quantities, and is likely to have a value of a few hundreds of kilometres/second over a fairly wide range of possible values for them. The density of the stream, therefore, is essentially determined by the rate at which mass is being exchanged between the two stars and the extent to which the stream diverges in the space between them. No assumptions have been made about the nature of the motions of individual particles within the stream, so this result is unaffected by whether the laws of particle dynamics or hydrodynamics apply. The only important assumption is that the stream is not used for storage. The density of disks cannot be estimated in this way, since it is possible that one function of disks is storage, but the method can be used in a system in which a disk exists, provided it is applied only to that portion of the stream between the mass-losing star and the disk.

For a stream such as has just been described, the area of its cross section, A, is approximately 1.5×10^{22} cm^2, and V_f is assumed to be 10^7 cm/sec, thus

$$n = (2/3)N\times10^{-29}/\text{cm}^3,$$

where N is expressed as a number of particles per second. Now a mass loss of $10^{-x} m\odot$/yr is approximately equivalent to a mass loss of

$$(1/3)\times 10^{-x-7} m\odot/\text{sec}$$

or

$$(2/3)\times 10^{26-x} \text{ g/sec.}$$

If all the matter lost is hydrogen (atomic weight $\sim 1.7\times 10^{-24}$ g) then

$$N = (2/3)\times(1/1.7)\times 10^{50-x} \text{ particles/sec}$$

and

$$n = (4/15.3)\times 10^{21-x} \text{ particles/cm}^3$$

or, approximately,

$$n = 0.25\times 10^{21-x} \text{ particles/cm}^3.$$

As an example, consider the case of U Cephei. Observations show that the period is increasing, and $\Delta P/P = 4.3\times 10^{-9}$. This increase can be interpreted as the result of mass transfer from the less massive star to the more massive. To obtain the amount of mass transferred, it is necessary to know the masses of each star. These are not well known for U Cephei. The primary has a spectral type of B6 to B8, and a mass of $6m\odot$ is not an unreasonable assumption, the mass ratio of the system is probably about one-half, so the secondary should be $3m\odot$. This is rather higher than is usually assumed, but the rate of mass transfer is more sensitive to the mass ratio than to the actual values of the masses. With these values of the masses, it is found that the period increase corresponds to a rate of mass exchange of about $10^{-6} m\odot$/yr. The expected stream density in the system U Cephei is, therefore,

$$\sim 0.25\times 10^{15} \text{ particles/cm}^3.$$

This is how the estimate in Table 12 was reached. It is a slight over-estimate, since the stream does not consist of pure hydrogen but undoubtedly contains a number of heavier particles. It is encouraging

that the estimate for the disk of U Cephei made by applying the Inglis–Teller formula to the hydrogen emission lines is also of the order of 10^{14} particles/cm^3. This suggests that a stream of the dimensions considered is not a bad approximation, and that it may be useful to introduce a *standard stream* having the characteristics that its final cross-sectional area is equal to that of a disk of solar radius, and that the final velocity of particles in it is 100 km/sec. Real streams in real systems can then be expressed in units of the standard stream.

CIRCUMSTELLAR DISKS

Many binary systems have emission features in their spectra which, as explained in an earlier section, are at least partly produced in rings or disks surrounding the primary components. Twenty-six systems are listed in Table 13, in order of increasing period. For twenty-one of these, the rotational velocities of the emitting structure are given. Most of these have been taken from a paper by Kruszewski (1967a), but the value for U Cephei (Batten, 1969) and unpublished values for ε Aurigae and VV Cephei have been added. The measurements made on the spectrograms of these last two (displacement of the emission peaks at the time of mid-eclipse) may not have the same physical significance as the measures of the displacement of emission lines seen in the spectrum of, for example, RW Tauri. Struve found an empirical relation between the velocity, V, of the emitting structure, and the binary period P of the form

$$V^3 \propto 1/P.$$

Huang and Struve (1956) showed that a ring consisting of particles moving independently under the gravitational influence of the primary star alone would obey the relation

$$V^3 = \frac{2\pi G m_1}{P}\left(\frac{m_1}{m_1+m_2}\right)^{1/2}\left(\frac{a}{a_{em}}\right)^3, \tag{1}$$

where a_{em} is the radius of the ring. If the factor

$$m_1\left(\frac{m_1}{m_1+m_2}\right)^{1/2}\left(\frac{a}{a_{em}}\right)^3 \tag{2}$$

TABLE 13. BINARY SYSTEMS WITH CIRCUMSTELLAR DISKS

System	Period (days)	Velocity of emission feature (km/sec)
WZ Sge	0.057	720
U Gem	0.177	670
DQ Her	0.194	500
SS Cyg	0.276	500
RU Peg	0.371	500
U Cep	2.5	310
RW Tau	2.8	350
U Sge	3.4	280
SW Cyg	4.6	
W Del	4.8	
AQ Peg	5.5	290
UX Mon	5.9	
S Vel	5.9	200
TT Hyd	7.0	270
WW Cyg	8.4	290
RY Gem	9.3	200
AW Peg	10.6	210
RS Cep	12.4	
DN Ori	13.0	
RW Per	13.2	220
RX Cas	32.3	150
SX Cas	36.6	150
KU Cyg	38.4	184
GG Car[1]	62.1	124
RZ Oph	261.9	142
AR Pav[1]	605	57
VV Cep	7450	63
ε Aur	9890	60

[1] I am indebted to Dr. A. D. Thackeray for drawing my attention to his result for AR Pavonis (Thackeray, 1959) after the completion of the manuscript. This result was not used to obtain equation (3) from which it deviates more strongly than that for any other system. In fact, AR Pavonis is closer to Struve's relation. Thackeray's figures for GG Carinae, given in the same paper, fit either relation.

does not differ widely from system to system, this relation explains the empirical result. Struve's result depended very heavily on the Algol-type systems and on a few long-period systems (SX Cassiopeiae, RX

Cassiopeiae, RZ Ophuichi, and ε Aurigae which does not fit the relation well). In these systems, the factor (2) probably is nearly constant. Observational results are now available for some of the very short-period ex-novae and systems of the U Geminorum type, although they are probably not very accurate. If these results are included, the best empirical relation (Fig. 9.3) is

$$4.6 \log V = 11.9 - \log P. \qquad (3)$$

Rings appear to be rotating more slowly in the short-period systems, and more quickly in the long-period systems, than Struve expected.

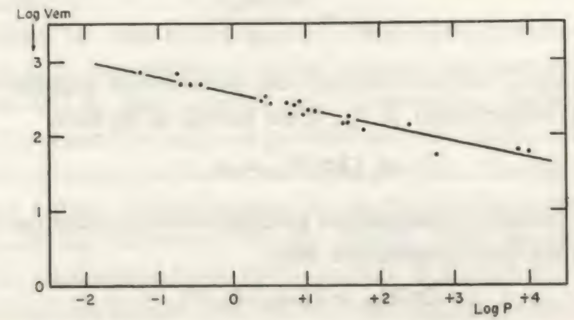

FIG. 9.3. Relation between V_{em} and orbital period for binary systems showing emission structures (see equation (3) in text).

Smak (1969) has pointed out that the measured velocity of emission lines necessarily gives an underestimate of the mean rotational velocity of the disk, the size of the error being dependent on the width of the disk in terms of its mean radius, and reaching as high as 20 per cent. If all the values of V have been underestimated by a constant fraction of V, the slope of the relation (3) will be unaffected. The range of masses encountered in the systems listed in Table 13 is close to a factor of 100: the components of SS Cygni have masses of about $0.2m_\odot$, those of VV Cephei have masses of about $20m_\odot$. It is unlikely that the factor (2), which contains m_1 explicitly, remains constant over this whole range, and this may partly account for the observed relation. On the other hand, the small scatter of the observed points about the

mean relation (about 0.1 in the logarithm) suggests that the relation may contain information about a fundamental property of circumstellar disks.

Kruszewski (1967a) has investigated in detail the possibility of forming disks about the primary component from matter ejected from the secondary star because its period of rotation is not synchronized with the orbital period of the system. The matter in the disk is supposed to have the same angular momentum per unit mass, h, as does the ejected matter that feeds it. This angular momentum is determined by the non-synchronism parameter, f, defined by

$$f = \frac{\omega - \omega_K}{\omega_K} \tag{4}$$

(where ω and ω_K are the observed and synchronous angular velocities of rotation, respectively). If P_{em} is the period of the disk,

$$a_{em}^2(2\pi)/P_{em} = h. \tag{5}$$

If the disk consists of independent particles moving under the influence of the primary star's gravitation only,

$$2\pi/P_{em} = (Gm_1/a_{em}^3)^{1/2}. \tag{6}$$

If the units of time, length, and mass are chosen as $2\pi/P_{em}$, a (not a_{em}), and m_1+m_2, then G is unity. If, further, $q = m_1/m_2$, then equations (5) and (6) reduce to

$$a_{em} = \frac{q+1}{q}\,h^2.$$

The radius of the disk can be predicted if q and h are known. The value of h can be calculated for any assumed value of f. Kruszewski found that no disk can be formed in a system in which the primary component (within the disk) has a fractional radius greater than 0.17, regardless of the mass ratio q, for any negative or plausible positive value of f. If the fractional radius is between 0.1 and 0.17, disks can only form if q is high, but for small fractional radii, such as are found for the primary components of SX Cassiopeiae, KU Cygni, U Geminorum, and DQ Herculis, a ring must form, whatever the mass ratio. The primary

components of RW Tauri, U Sagittae, and U Cephei all have fractional radii greater than 0.17, and yet disks have been observed in them, at least intermittently. Kruszewski's results probably can be applied to any low-speed ejections of matter from the secondary component (5 km/sec–10 km/sec). At least some circumstellar disks, therefore, cannot be explained if the particles in gas streams move independently of each other, although those in the three systems mentioned are probably unstable. Perhaps the ring forms only when the secondary star is particularly active. Gorbatskii (1969) finds that the ring in U Geminorum would collapse in about a month if the supply of material were cut off.

The failure to take collisions and interactions between particles into account in computations of gas streams has been criticized by many authors. At the densities and temperatures deduced for the streams, the mean free paths of particles are very short—about 60 cm for neutral particles, and 4 cm for charged particles. (Earlier statements in various papers, including one by Batten (1970), quoting much longer paths, are incorrect.) Huang (1966b, 1968) has developed a method in which departures from particle dynamics are treated as perturbations and he finds that it is easier to form rings when collisions are taken into account. Hydrodynamic treatments have been attempted by Prendergast (1960), Kitamura (1970), Sobouti (1970), Kříž (1970), Korovyakovskii (1970), and Biermann (1971). Of these, the last named seems to me to be the most important. Biermann has emphasized that the gas flow in streams must be supersonic through most of the length of the stream. The velocity of sound, c, in a gas is given by

$$c = \sqrt{\gamma RT}$$

where γ is the ratio of specific heats (5/3 in the present case), R is the gas constant, and T the temperature. It is difficult to define the temperature of the stream, because conditions in the stream must be very different from thermodynamic equilibrium, but the mean energy of the atoms must correspond to temperatures between 1000°K and 10,000°K so c is about 10 km/sec. Even if the stream leaves the secondary component with about this speed, it is quickly accelerated by the

gravitational attraction of the other star, its motion becomes supersonic. This makes the problem of treating the motion a little easier, because in supersonic flow there is less likelihood of turbulence, even though the mean free paths are very short, and collisions between particles are inevitable. Some investigators have thought that turbulence is inevitable in the streams, because the observed densities and velocities imply very high values of Reynolds' number (Crawford and Kraft, 1956; Kopal 1958). Biermann has pointed out, however, that at supersonic velocities the boundary conditions are very important in determining whether or not the flow will be turbulent. Liepmann and Roshko (1957) explain that turbulence can be "radiated" away in a supersonic flow.

The chief result of the particle-dynamics computations—the existence of a large stream conveying matter from one star to the other—is confirmed by these more accurate calculations. It is, indeed, required by the large-scale mechanics of the situation, and it appears to be supported by the observations, and to be in accord with the theory of evolution of binary systems elaborated in the next chapter.

EFFECTS OF CIRCUMSTELLAR MATTER ON ORBITAL ELEMENTS

Three possible kinds of effect of circumstellar matter on orbital elements can be clearly distinguished. First, there are the apparent effects caused by the blending of the spectrum of stream or disk with that of the star, or by the interference of the stream and disk with the light curve. These effects are discussed in Chapters 1 and 8, and are not treated further here. Second, there may be changes in the orbital elements caused by the resistance offered by the circumstellar matter to the motions of the two stars. These effects are discussed, in other connections, in Chapters 5 and 6, and are considered only briefly in this particular application here. Third, the process of mass transfer affects the relative motions of the two components of a binary system and therefore changes the orbital elements. These effects are also discussed in Chapter 5. They are the most important for the evolutionary history of the whole system and are further discussed in this section.

Motion through a resisting medium may decrease the period, major semi-axis, and eccentricity of an orbit (Smart, 1953), and have a negligibly small effect on the element ω, as discussed in Chapter 6. The theory of the frictional resistance offered by a cloud to a star is not fully developed. Huang's approximate formula for the change in period is given as equation (2) of Chapter 4. From it was deduced the result that two stars like the Sun moving through a cloud of density 10^{11} particles/cm^3 should suffer an easily detectable continuous decrease in period of one part in 10^8 or 10^9. The density of 10^{11} particles/cm^3 was chosen because it is the "mean" density found for clouds from the figures of Table 12. This is a misleading mean, however, being made unduly high by one or two systems with exceptionally dense clouds. On the other hand, the interface at which the frictional resistance acts may be that between the cloud and disk. Since this interface has a much larger area than does the surface of the star, frictional resistance would be higher, for a given cloud density, than the value computed in Chapter 4. The observational fact that there is no known case of a continuous period decrease that can be attributed to the effects of a resisting medium suggests that in many systems the cloud densities are rather lower than the figures given in Table 12.

The effect of mass transfer on the orbital period is given by equation (1) of Chapter 4, repeated here, together with the related equation for the semi-axis.

$$\frac{\Delta P}{P} = 3\,\frac{2\mu-1}{\mu(1-\mu)}\,\frac{\Delta m}{m}, \tag{7}$$

$$\frac{\Delta a}{a} = 2\,\frac{2\mu-1}{\mu(1-\mu)}\,\frac{\Delta m}{m}. \tag{8}$$

The meaning of the symbols, other than a and P, is explained on p. 97. These equations apply if mass and angular momentum are conserved in the process of mass transfer. Mass and orbital angular momentum will clearly be conserved if mass is exchanged between the components

without any being lost from the system. The fate of rotational angular momentum is not so obvious, but even in the W Ursae Majoris systems the rotational angular momentum amounts only to about one per cent of the orbital angular momentum (Smak, 1964b), so it is not very important. Equations (7) and (8) are not valid if substantial amounts of mass are lost from the system. Nevertheless, they have been found in practice to give considerable insight into the course of evolution of close binary systems.

The orbital eccentricity is not greatly affected by a slow, steady process of mass loss. Kruszewski finds that, in an initially circular orbit,

$$\frac{e\,\Delta e}{1-e^2} = 0,$$

and for very small values of e, Δe is proportional to the mass lost or transferred. Piotrowski (1964b, 1967) considered an orbit of initially small eccentricity in which mass is exchanged only when the stars are at periastron (see also Martin, 1962). The absolute sizes of the Roche lobes vary for stars moving in an elliptical orbit, and they are smallest at periastron; and a star that nearly fills its lobe may begin to lose mass then. Piotrowski found that for most possible trajectories the eccentricity is reduced. If it is increased, so also is the semi-axis. If mass is lost from the system very rapidly, the eccentricity can be more drastically changed and the binary system may even be disrupted (Hadjidemetriou, 1967). The mass lost in a nova outburst is probably insufficient to achieve this, but if a binary component were to become a supernova the system might be disrupted.

The element ω is also affected by mass transfer. It will advance at a rate proportional to the rate of mass transfer. It also depends, in a complicated way, on the orbital eccentricity, angular momentum, and the speed of ejection of the mass. If the mass exchange is slow ($\sim 10^{-5} m_\odot$/year or less) and the ejection speeds are only a few kilometres per second, then the advance of ω is likely to be negligible compared with that produced by other causes, unless the eccentricity is very small (in which case ω cannot be well determined observationally).

The variety of the possible speeds and trajectories of matter being transferred is infinite, and it is impracticable to give general formulae for the effects of mass loss and mass transfer on the orbital elements. The pioneer papers in the field are those of Kuiper (1941), Wood (1950), and Huang (1956). More detailed investigations have been made by Piotrowski (1964 b, c, 1967) and Kruszewski (1964b, 1966) upon whose results I have drawn very heavily in this section. The last reference is a useful review paper of the whole field.

CHAPTER 10

EVOLUTION AND ORIGIN OF BINARY SYSTEMS

THEORY OF EVOLUTION OF SINGLE STARS AND ITS APPLICATION TO BINARY STARS

In this section, current ideas of the evolution of single stars are briefly summarized. I have not attempted to present an authoritative or complete review. The reader is referred to the many books and reviews on this subject for a more detailed description. The evolution of a star can be presented in terms of either its radius or luminosity. Figure 1.1 illustrates the changes in luminosity typical for an evolving star. For reasons that will shortly become apparent, attention is directed here to the changes of radius induced by evolutionary processes. The star first condenses from the interstellar medium as an object with a very large radius. It contracts fairly rapidly, until its centre is hot enough for nuclear reactions to begin. The star then settles down on the main sequence of the Hertzsprung–Russell diagram in a position determined by its mass, and to a lesser extent its chemical composition. The star stays on the main sequence with a constant radius and constant luminosity until a large part of the hydrogen in its central core has been burnt into helium, when it begins to derive energy from the contraction of this central core. The luminosity and radius then begin to increase slowly. According to figures derived by Plavec (1968a) from diagrams published by Iben in several papers [and summarized by Iben (1967) himself], stars with masses between one and fifteen solar masses will approximately double their radii during this stage. A small contraction follows (except for stars of about the solar mass) as hydrogen begins to burn in a shell surrounding the core. This hydrogen is rapidly depleted, however (there was not very much of it), and a rapid expansion of the star begins, so that it becomes a red

giant. During this phase, the radius may increase by a factor of between ten and a hundred, the precise value depending on the mass of the star. The star then reaches the greatest radius it has at any stage for which detailed calculations have yet been made. Thereafter the radius decreases, and then fluctuates, but its subsequent behaviour is not well known. It is possible that stars of some masses, as they successively exhaust other nuclear fuels, attain even greater radii, but this is not yet known. The rate at which a given star progresses through these phases depends critically on its mass. The more massive the star, the faster its evolution. It is well known, for example, that the main-sequence phase of a massive star of spectral type B lasts at most for 10^7 years, while a star like the Sun should stay on the main sequence for between 10^9 and 10^{10} years.

To see the application of these ideas to the evolution of binary systems, the definition of a close binary system that was given in Chapter 1 should be recalled. A binary system is to be considered close if the components can affect the course of each other's evolution. It was pointed out in Chapter 1 that this definition directs attention to the whole history of the component stars, and that the stars can be expected to affect each other when the radius of either one of them exceeds that of the Roche lobe. This can happen before the star reaches the main sequence, or after it leaves that sequence. Plavec (1968a) has suggested that virtually all binary systems contain components that will interfere with each other in the pre-main-sequence phase of contraction, and that an unknown amount of mass is probably exchanged during this stage. No detailed calculations have been made, however, although some discussion of the role of mass exchange in this phase has recently been published in connection with the formation of W Ursae Majoris systems (Whelan, 1970). After the main-sequence phase, the theory predicts that the more massive star will expand, and therefore fill its Roche lobe, first. Plavec has shown that the differential rate of evolution for two stars is quite sensitive to the difference in mass, and in almost all systems the most massive star should fill its Roche lobe first. The only possible exception is in systems containing stars of about $2.5m_\odot$ each, one slightly above this limit, the other

below. Just at this point in the main sequence, the less massive star could fill its smaller Roche lobe first, even though its expansion began later. Once a binary component fills (or more realistically slightly overflows) its Roche lobe, it becomes unstable, and must begin to lose mass. The previous two chapters are concerned with the immediate fate of that mass. If the mass returns to the star, the instability continues, and eventually the mass must either be transferred to the companion star, or lost to the system. It is shown in Chapter 9 that either of these processes changes the orbital elements. Thus the evolution of stars within a binary system is inextricably woven into the evolution of the system itself.

Thus, it is not obvious whether a given system is close or wide, unless it is possible to calculate the radii of its components over the whole lifetime. The ζ Aurigae systems, for example, at first sight seem to be wide. The giant or supergiant components have presumably gone through most of their rapid-expansion phase, and they have not yet filled their Roche lobes. It is not known, however, what subsequent expansion is in store for them, and when the secondary components expand they, being less massive, will have smaller Roche lobes to fill. The system of VV Cephei, the primary of which may not fill its Roche lobe, certainly shows signs of interaction between the components, and must be considered close. Maybe the natural instability of giant stars leads to interaction between components even in systems that would not normally be considered close. On the other hand, as is pointed out in Chapter 1, stars of very low mass may never expand from the main sequence, and although binaries containing them might have very short periods, the systems may not be close, in the sense used in this book.

Plavec (1968a, 1970b) has worked out minimum periods for binaries containing stars of given mass to be "wide". He finds that even components of binaries with periods of a few years may affect each other in the advanced stages of stellar evolution when one of the components has become a supergiant. He concludes that virtually all spectroscopic and eclipsing binaries, and some visual binaries are "close". Even Sirius, with its period of fifty years, has been considered as a possible

end-product of mass exchange (Lauterborn, 1970). Indeed its white-dwarf companion strongly supports this idea, because in certain circumstances mass exchange can produce a star with the properties of a white dwarf (see p. 237).

Computations of the course of stellar evolution are usually made by Henyey's method in which the time-dependent equations of stellar structure are expressed as difference equations and solved by iteration (Henyey *et al.*, 1959, 1964). Thus the continuous evolution of the star is described by a series of models computed for suitable time intervals. When a star begins to lose mass because it has filled its Roche lobe, the method of calculation must be modified somewhat to take account of the change of mass with time. Details of these modifications are given in the original papers, and need not be repeated here. The periods of evolution in which mass loss is expected, are the periods of expansion of the star. The period of slow expansion, during which the radius only doubles, can lead to loss of mass if the initial separation of the components is small enough. This has come to be known as mass loss in phase A. If a star does not fill its Roche lobe during this time of expansion, it is very likely to do so during the rapid expansion, and mass loss at this stage is known as mass loss in phase B. Some authors have used the nomenclature I and II for these phases. There is an increasing interest in the possibility of mass loss in a third phase of expansion, after the exhaustion of helium in the core, known as phase C. Since stars of a solar mass or less do not have a time of contraction separating the two phases of expansion, the distinction between the two phases is not so obvious for these stars. Nevertheless, the internal structure of the expanding star is still different in the two phases. The difference in the rates of expansion, for more massive stars, is best illustrated by a simple numerical example. A star of $5m_\odot$, according to Plavec (1968a), increases its radius, during the slow expansion, from 2.4 to 4.4 $R_\odot$ in 6.56×10^7 years. That is a rate of expansion of about 6.8×10^{-5} cm/sec. During the rapid phase, the radius of the star increases by a further $68R_\odot$ in 4.7×10^6 years. This is a rate of expansion of about 3.2×10^{-2} cm/sec.

THE ALGOL PARADOX

One particular class of binary systems, the Algol-type systems, presents a paradox when considered from the point of view of the theory of evolution outlined in the previous section. These systems contain a main-sequence primary component of spectral type between about B5 and A5, and a larger subgiant secondary star, usually of spectral types G or K. In most systems this subgiant fills its Roche lobe, but there is a significant subgroup in which the secondary, although clearly a subgiant, does not fill the lobe.

As pointed out in Chapter 5, the masses of the stars in Algol-type systems are not well known. The radial-velocity curve of the primary component is frequently distorted in the ways discussed in Chapter 8, and the mass ratio is hard to determine because the secondary component is usually invisible except during the eclipse of the primary. The few radial velocities that can be determined for the secondary at these phases are not suitable for a very accurate estimate of the mass ratio. Some inferences can be made about the masses, however. Struve (1948) pointed out that the mass functions of these systems are usually low. Thus, if the primary is assumed to be normal for its spectral type, the mass of the secondary must be smaller than that of the Sun, although its spectral type is similar and its radius several times greater. Parenago (1950) used similar arguments to show that these secondary components are subgiants, overluminous and oversized for their small masses. Independently, Kopal (1955) and Crawford (1955) found that these subgiants, for the most part, fill their Roche lobes [a conclusion anticipated by Wood (1950) in his work on period fluctuations] and both surmised that the stars had expanded to fill these lobes. The paradox of the Algol-type systems consists in the fact that on all reasonable hypotheses about the mass ratios of the systems and the masses of the primary components, the expanded subgiant is by far the less massive star of the pair.

RESOLUTION OF THE ALGOL PARADOX

The resolution of the Algol paradox was first suggested by Crawford in his original investigation of the properties of subgiants. His idea was that the apparent secondary of the Algol-type systems had originally been the more massive star of the pair. It did expand first, and spilled its matter over to the less massive companion. This process reversed the mass ratio and led to the state of affairs now observed: an undermassive, overluminous, evolved star accompanied by a more massive, apparently unevolved star.

At the time that Crawford wrote (1955), only the outlines of the present theory of stellar evolution existed. Only the initial departure from the main sequence had been computed in any detail, and it was not possible to submit Crawford's hypothesis to a quantitative check. Naturally, it raised several questions: why is the spectacular transfer of mass from one component to the other not observed? (Perhaps 80 per cent of the mass must be transferred to reverse the mass ratio.) Will the star that receives this mass continue to look like a normal main-sequence star of its new mass? Will the mass really be transferred, or simply lost to the system? What is the chemical composition of the newly exposed surface layers?

The next step was taken by Morton (1960) who showed that the reduction in mass ratio during the process of mass exchange would lead to a shrinking of the Roche lobe, thus increasing the rate of mass exchange, once the process had begun. Morton also found that if mass exchange started after a star had already left the main sequence, the removal of mass would not decrease the radius, so mass would continue to flow away very rapidly. The process takes place on a Kelvin time scale, that is in about 10^5 years for a star of $10m_\odot$. Smak pointed out (1962) that Morton had failed to take account of the changes in the orbital elements that mass transfer must inevitably produce. By hypothesis, the more massive star loses mass to the less massive star, until the mass ratio is reversed. As is shown in Chapter 4, the period and major axis of the orbit must therefore decrease until the masses are equal. The absolute size of the Roche lobes of each star

also decreases, and Morton's argument is actually strengthened when this correction is made to it.

The major breakthrough was made by Kippenhahn and Weigert (1967) who investigated the process of mass exchange and its effect on stellar evolution. Similar calculations were made almost simultaneously by Paczynski (1966) and shortly afterwards many important results were obtained by Plavec and his associates (references to these are given as the results are used in other parts of this chapter). Kippenhahn and Weigert were the first to distinguish the two possible phases (A and B) of expansion in which mass transfer can occur. They computed what would happen in a binary containing a primary of originally $9m_\odot$. Whichever phase of expansion the mass exchange occurred in, they found that most of the mass was exchanged very rapidly. For expansion in phase A, the mass is reduced from $9m_\odot$ to $3.73m_\odot$ in 6×10^4 years. The original secondary (assumed to have a mass of $5m_\odot$) becomes the more massive star, but the original primary continues to lose mass very much more slowly ($\sim4\times10^{-8}\,m_\odot$/year for about 10^7 years). Mass transfer in phase A (before the complete exhaustion of core hydrogen in the star) thus leads to a first very rapid transfer of mass, unlikely to be observed because of its short duration, followed by a prolonged slow transfer of mass. During this phase, the mass-losing star, which has now become the less massive star of the pair, fills its Roche lobe, and the system is semi-detached.

If the mass is transferred from one star to the other after the core hydrogen of the original primary has all been burnt, that is during the rapid phase B of expansion, the course of events is rather different. Kippenhahn and Weigert calculated the course of evolution for a pair of $9m_\odot$ and $3.13m_\odot$ with mass transfer in phase B. They found that the mass of the original primary is reduced to $2m_\odot$ after 4×10^4 years. After this the original primary begins to contract, and becomes a helium star. There is no prolonged phase of slow mass transfer in this situation.

These results strongly suggest that the Algol-type systems are formed by mass exchange during the phase A expansion of the original primary component. The particular example chosen by Kippenhahn and Weigert, however, is much more massive than an Algol-type system. Subsequent calculations by Plavec and Paczynski for less massive systems have indeed confirmed that the properties of Algol-type systems can be at least qualitatively explained by this hypothesis. The paradox of the Algol-type systems has therefore been resolved in the manner originally suggested by one of its discoverers.

Although the new theory suffices to give a qualitative representation of the Algol-type systems, the detailed quantitative comparison has not proved fully satisfactory. Thus although the theory predicts that the transformed primary will be an overluminous subgiant, the precise values of radius and luminosity obtained are not in accord with the observations, being in general too small (Paczynski, 1967b). Paczynski and Ziółkowski (1967) have shown that better agreement between theory and observation may be obtained if it is assumed that about half the mass ejected is lost from the system altogether. Plavec and his group have emphasized the importance of the chemical inhomogeneity (or age) of the primary star at the onset of mass exchange, and of the role of the initial mass of the secondary component. The observed masses and absolute dimensions of stars in Algol-type systems are very uncertain, and as Plavec (1970b) has pointed out, only a relatively small proportion of the possible number of models has actually been computed, and complete agreement between theory and observation is unlikely to be obtained until improvements in both have been made.

There is an increasing tendency to distinguish between two groups of semi-detached systems: those of small total mass ($<2.5m_\odot$) in which the present secondary components are very overluminous for their rather small masses ($<1m_\odot$), and those of larger total mass (up to about $3m_\odot$) in which the overluminosity of the present secondary is much less marked (Paczynski, 1967a; Giannone *et al.*, 1968). Recent calculations by Refsdal and Weigert (1970) show that the first-named group probably evolved by mass exchange in phase B of the original primary's evolution, although Ziółkowski (1968) suggests that they have formed as a result of mass exchange that began when the primary star was in phase A of its expansion, but which continued until it

reached the phase-B expansion. It will be remembered that for stars of approximately the solar mass there is no clear distinction between phases A and B of expansion, although the two phases can be distinguished by the exhaustion of the core hydrogen, just as for massive stars. Thus if mass transfer is not completed until after the core has entered phase B, the end product must be different. This case of mass transfer has come to be known as case AB, and has also been studied by Horn (1970).

ENERGY FOR THE MASS TRANSFER

The rate at which mass is transferred from one star to the other depends primarily on the initial mass of the original primary star. The more massive the star, the faster it will lose mass, although other factors (e.g. the age of the primary, and the mass ratio of the system) also affect the rate. Paczynski (1967c) has calculated models for very massive systems in which the peak rate of transfer is about 10^{-3} $m\odot$/year. Even in systems of quite moderate mass, the maximum rate during the period of rapid mass transfer is of the order of $10^{-5} m\odot$/year. The observed period change in β Lyrae can be accounted for by a mass-transfer rate of about $10^{-4} m\odot$/year (Chapter 4). In this section, some of the processes that may be involved in the removal of this mass from the primary star are described, and the energy they require is estimated.

Three processes of mass ejection have been proposed:

(i) ejection of particles from the Lagrangian point, as a result of their thermal diffusion, at velocities $\leqslant 10$ km/sec;

(ii) ejection of particles from the same point, at similar velocities, because the orbital and rotational angular velocities of the star are different;

(iii) ejection at high speeds (~ 100 km/sec) from the Lagrangian point by some unspecified mechanism.

Whether the initial ejection speed is low or high, the ejected stream will rapidly be accelerated, as explained in Chapter 9, and the observed velocities of these streams should be several hundred km/sec, as found.

Ejections of type (i) have been studied by Kopal (1959f) and Plavec *et al.* (1964). The hypothesis of thermal diffusion is attractive because it requires no source of energy, other than the thermal energy of the stellar atmosphere, and the observed stream densities of about 0.1 per cent of the density of the ejecting star's atmosphere are probably about what is to be expected. The maximum possible rate of ejection by this means can be estimated with the help of a fairly crude model (Fig. 10.1). Plavec, Sehnal, and Mikuláš showed that particles can only be ejected at or near the Lagrangian point. Consider a surface,

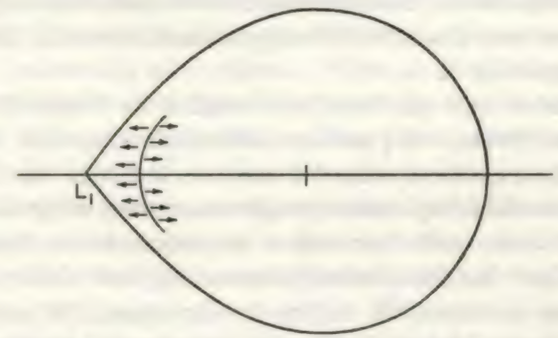

Fig. 10.1. Model for estimating rate of mass transfer from thermal diffusion. Half of the particles at the indicated surface are supposed to possess the mean thermal velocity directed outwards.

centred on this point, whose area is 1 per cent of the total surface area of the star. Let the mean particle density at this surface be n/unit volume and suppose that half these particles have exactly the mean thermal velocity, V, outwards, and the other half exactly the velocity V inwards. If the area of the surface is A, the number of particles escaping in unit time is

$$\tfrac{1}{2}nAV,$$

where

$$V = \left(\frac{8RT}{\pi\mu}\right)^{1/2}$$

and μ is the mean molecular weight, R the gas constant and T the temperature. Although the temperature is very difficult to define in this part of the atmosphere it probably lies in the range 1000°K to 10,000°K, and the expected value of V is about 10^6 cm/sec. In a late-type subgiant atmosphere n should be between 10^{16} particles/cm^3 and 10^{17} particles/cm^3. An undistorted subgiant of radius $4R\odot$ has a total surface area of about 10^{24} cm^2, so A is 10^{22} cm^2 and

$$\begin{aligned} nAV &\approx 10^{45} \text{ particles/sec} \\ &\approx 10^{21} \text{ gm/sec (for hydrogen)} \\ &\approx 3\times10^{28} \text{ g/yr.} \end{aligned}$$

Thermal evaporation, therefore, can transfer mass at rates up to about $10^{-5}\,m\odot$ per year (because of the approximations made, this is probably an upper limit).

Ejections of type (ii) have been studied by Kopal (1959f) and Kruszewski (1964a, 1967) and are discussed in Chapter 9. The possibility of these ejections arises because it is supposed that in normal close binaries tidal friction has brought the rotation of the components into synchronism with their orbital revolution. A star that is losing mass because it has expanded must be rotating more slowly, if angular momentum was conserved during the expansion. The non-synchronism parameter f [defined in Chapter 9, equation (4)] must be negative, but will probably not differ very much from zero. The angular velocity ω_K is $2\pi/P$, and the corresponding linear velocity at the surface of the star (radius R_*) is

$$V_K = (2\pi R_*)/P.$$

The actual linear velocity at the equator of the star is

$$V = (1-f)(2\pi R_*)/P$$

and the velocity of ejection $V_e = |V - V_K|$ or

$$V_e = |(2\pi f R_*)/P|.$$

For $|f| = 0.1$, $P = 26\times10^4$ sec (3 days), $R_* = 2.8\times10^{11}$ cm ($4R\odot$),

$$V_e = 0.7\times10^6 \text{ cm/sec.}$$

Ejections caused by non-synchronism are therefore also likely to be low-speed ejections, as assumed in Chapter 9. Particles will be ejected from about the same level and the same area as was the case for thermal diffusion. Therefore the maximum mass-transfer rates possible by these two processes are similar. No energy is required for ejections caused by non-synchronism except the rotational energy of the star, but the process cannot sustain itself because the rate of expansion of the star and the change in size of the Roche lobe tend to decrease the difference between the actual and synchronous rotational velocities.

Low-speed ejections of these types cannot convey mass from one component to the other sufficiently rapidly in the more massive binary systems. In Chapter 9 it is also shown that they cannot account for the disks observed in some systems. Plavec *et al.* (1964) and Plavec and Kříž (1965) have investigated the trajectories followed by particles ejected at higher speeds ($\approx$100 km/sec). The obvious analogy that comes to mind is that of mass loss from the Sun through prominence activity. Eruptive prominences (surges and sprays) associated with bright flares have been observed to eject matter from the Sun at speeds up to twice the velocity of escape (Kleczek, 1964). A bright flare can eject about 10^{16} grams at this speed (i.e. 1200 km/sec), and its total energy is of the same order as its kinetic energy, namely 10^{32} ergs (Sweet, 1969). The optical part of the flare has a similar density to that found for the stream, but the ejecta probably come from regions of lower density (Menzel, 1968). Flares as large as this are rare, but the velocities need not be as large as 1200 km/sec. Plavec and Kříž find that the most favourable range of *ejection* speeds for ring-like trajectories is 100 km/sec to 200 km/sec. Only 8 per cent of eruptive prominences observed on the Sun have velocities in excess of the escape velocity, but about 25 per cent have velocities in this favourable range (based on information in Kleczek, 1964). Near the Lagrangian point such prominences would escape to the other star. The kinetic energy taken by each gram of matter that leaves the star at a velocity of 100 km/sec is 0.5×10^4 ergs. A mass-exchange rate of $10^{-6}m\odot$/year (well below the peak rate even for systems of moderate mass, and corre-

sponding to the observed period change in U Cephei) is equivalent to approximately 6×10^{19} g/sec, and at high speeds would take with it about 3×10^{33} ergs/sec. This is comparable to the total radiant energy of the Sun or of an Algol-type subgiant (a star of effective temperature 4100°K and radius $3R_\odot$ would radiate about 9×10^{33} ergs/sec). Ten "class 3" flares would be needed every second *at the Lagrangian point* to supply the energy taken by the escaping matter, but they would eject less than 1 per cent of the required mass. Judged as a mechanism for mass transfer, solar flares are very inefficient.

High-speed ejection must take place in some systems at some stages, because the low-speed mechanisms cannot transfer mass quickly enough. At these stages, the mass-losing star no doubt has the energy needed, but the subgiant secondaries of Algol-type systems do not. The rings observed in these systems, therefore, cannot be formed by matter moving in high-speed trajectories. At the present stage of their evolution, Algol-type systems are probably not losing much matter into space, because the mass transfer is being effected at low speeds. Earlier, in their rapid-transfer phase, mass may have been lost from the systems.

The energy needed by some systems for high-speed ejections cannot come from the increasing potential energy of expansion of the mass-losing star. As mentioned earlier in the chapter, even in the phase of rapid expansion, a star of $5m_\odot$ increases its radius only by $\Delta r = 3.2 \times 10^{-2}$ cm/sec. The increased potential energy of the stellar atmosphere is $mg\Delta r$, where m is the mass of the atmosphere and g the acceleration due to gravity. According to Allen (1963) the value of g for a main-sequence star of $5m_\odot$ is about 10^4 cm/sec^2, and there are 10^{23}/cm^2 atoms above the photosphere. If these atoms are all hydrogen, and the radius of the star is $2.4R_\odot$, the mass of the atmosphere is about 10^{23} g. Thus the rapid expansion increases the energy of the atmosphere by only 3×10^{27} ergs/sec. Although some of these figures must be changed slightly, even before the rapid expansion begins, the increase of potential energy is many orders of magnitude too low to drive the mass transfer. The expansion, rather, triggers a complete loss of thermal equilibrium in the mass-losing star because

of the loss of the outer layers (Plavec, 1968a). The energy to drive the rapid mass exchange comes from the radiant energy of the star.

This is illustrated in Fig. 10.2 which shows the track on the Hertzsprung–Russell diagram of a star losing mass. Its initial mass was $5m_\odot$, and mass loss began (at b) when the hydrogen content in the core, X_c, was 0.25. When mass loss begins, the star's luminosity suddenly decreases, and this continues, even after the masses of the two components become equal (at c: the original mass of the secondary

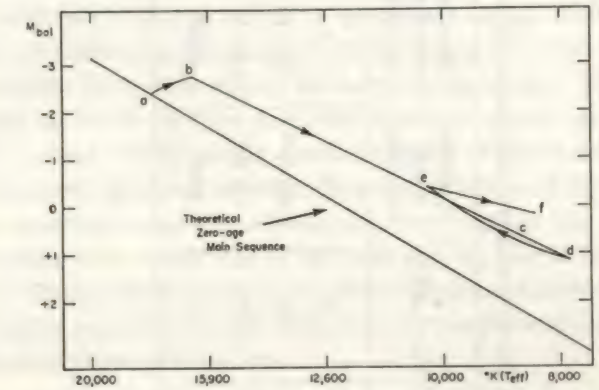

FIG. 10.2. Evolutionary track in the Hertzsprung–Russell diagram for a star of $5m_\odot$ that begins to lose mass to a companion of $3m_\odot$ when the core hydrogen content $X_c = 0.25$ (at b). The star loses mass and luminosity very quickly. Masses of the two components are equal at c, 8800 years after mass loss began and the primary component reaches its minimum luminosity at d, 11,900 years after mass loss began. Mass loss continues more slowly from d to e for another 581,000 years, but luminosity is recovered. Mass loss finishes at f, 65 million years after point e was reached. The slow mass loss from e to f corresponds to the Algol phase. (Data from Plavec *et al.*, 1969.)

was $3m_\odot$) until a minimum is reached at d. The star then increases in luminosity until the point e (often called the "end" of rapid mass exchange) after which it becomes fainter again as slow mass exchange continues. In Table 14 the following data are shown for this system: time elapsed from b; mass of primary star; mass lost by primary star; logarithm of the luminosity of the primary star relative to that of the

TABLE 14. ENERGY AVAILABLE FOR MASS TRANSFER IN SYSTEM CONTAINING STARS OF $5m\odot$ AND $3m\odot$

Stage	Time (10^3 yr)	m_1 ($\odot$)	Mass lost ($\odot$)	Log $L_*/L\odot$	Log $L_s/L\odot$	$(L_s - L_*)/L\odot$	Available energy (ergs/sec)	Rate of mass loss ($m\odot$/yr)	Average energy/g (ergs)	Ejection velocity (km/sec)
b	0	5	0	2.9	2.9	0	0	0	0	0
c	88	4	1	1.6	2.6	360	1.4×10^{36}	1.9×10^{-5}	1.2×10^{15}	490
d	119	3.5	1.5	1.4	2.3	175	7×10^{35}	1.3×10^{-5}	0.85×10^{14}	130
e	700	2.6	2.4	2.0	1.8	−37	none	small	small	small

Note 10^{-5} $m\odot$/yr $= 0.63\times10^{21}$ g/sec.

Sun; logarithm of the luminosity of a single star of the same mass and value of X_c (rapid mass loss is so quick that X_c does not change during this phase) also expressed relative to that of the Sun; the difference between these two luminosities (in solar units); the equivalent energy in ergs ($L\odot = 3.86\times10^{33}$ ergs/sec); and the rate of mass loss in units of 10^{-5} $m\odot$/year. All these data can either be found or readily derived from the work of Plavec *et al.* (1969) except those in column 6, which are taken from the companion work of Horn *et al.* (1969). The energy indicated in column 8 must be taken away by the escaping mass. No doubt much of it is locked in as radiant energy of this matter, but for an order-of-magnitude calculation it can be assumed that it is all available for kinetic energy of the escaping matter. The last two columns of Table 14 then give the available energy per gram of matter and the corresponding ejection velocity. There is ample energy for high-speed ejections between the phases *b* and *e* of the mass loss. Note, however, that the mass-losing star regains the luminosity appropriate to a single star of its mass and X_c before reaching the point *e*. There is thus no energy to drive high-speed ejections for some time before point *e* is reached and the star can be said already to be in the slow phase of mass loss.

WIDER IMPLICATIONS OF MASS EXCHANGE

Although work on the theory of mass exchange in close binary systems was stimulated by the desire to resolve the Algol paradox, the chief interest of Kippenhahn and Weigert was the problem of the origin of white dwarf stars. The observed number of these stars is too high for them all to be explained as the remnants of novae or supernovae, and the process of mass exchange seemed to offer another way to produce such objects. The remnant of the original star, in many cases, does turn out to be a degenerate star of low mass, i.e. a white dwarf. These objects can be produced when mass exchange takes place after the exhaustion of hydrogen in the core (case B) if the original mass of the primary star was too small to permit helium burning to begin in the remnant. These white dwarfs are always of low mass (less

than about $0.5m_\odot$) since more massive remnants become helium-burning stars. Reference is made in an earlier section to Lauterborn's attempt to produce a more massive white dwarf (about $1m_\odot$) such as the companion to Sirius. His result is that if mass exchange takes place in phase C (after the end of helium burning in the core) a white dwar of the required mass will be produced. Mass exchange can only take place in this phase if the system has a much larger initial separation than most of those for which models have been calculated, because the primary has survived transformation into a red giant without filling its Roche lobe. Lauterborn considers a system consisting initially of stars of $5m_\odot$ and $2m_\odot$, with a separation of $302R_\odot$ and an orbital period of $0^y.63$. After mass exchange in phase C, the separation is increased to $815R_\odot$, and the period to $2^y.8$. The original primary has become a white dwarf of $1m_\odot$; the other star a main-sequence star of $6m_\odot$. This system inevitably calls Sirius to mind, but the separation of the components of Sirius is much greater than that of these stars. The eccentric orbits of the components of Sirius make precise calculation difficult, but it seems likely that the system has undergone mass exchange in phase C. The gravitational binding of systems like Sirius is weak, and could be broken by the process of mass exchange. This would provide a mechanism for the formation of unaccompanied white dwarfs.

The system of Sirius also provides the starting point for another possible application of mass-exchange theory. Sirius A displays some of the characteristics of Am stars in its spectrum (Conti, 1965), and since there is evidence that the binary frequency among Am stars is considerably higher than average (Chapter 2), it is natural to try to link the Am characteristics to binary nature, and possibly to mass exchange. This step has been taken by Conti, and elaborated by van den Heuvel (1967, 1968 a, b, c, d) who have suggested that all Am stars originated by mass exchange and are accompanied by white dwarfs. A similar suggestion had already been made by Fowler *et al.* (1965) in regard to the Ap stars, and this has also been developed by van den Heuvel in the same papers and by Guthrie (1969). Van den Heuvel sees both Am and Ap stars as the end products of mass exchange. He

believes that the Ap stars are the transformed secondaries after they have received mass from an original primary of at least $2.8m_\odot$ by mass transfer in phase B. The Am stars are supposed to be the results of mass exchange in phase C (consistent with Lauterborn's work, although van den Heuvel's studies were the earlier). In both cases, it is supposed that the original primary has become a white dwarf. The low percentage of detected spectroscopic binaries among Ap stars is explained by supposing that the transformed orbital parameters and mass ratio are such as to make the velocity variations very small, or even that the binary system has been disrupted during mass exchange. The high percentage of detected spectroscopic binaries among the Am stars is explained by assuming that the transformed orbital parameters and mass ratios favour discovery. Although van den Heuvel builds a strong circumstantial case for his thesis [the Am and Ap stars must be evolved, evolved binaries with spectral types in this range must have characteristics shown by these stars, "blue stragglers" in clusters can be explained as binary systems in which mass exchange has taken place, mass exchange will bring chemically anomalous layers to the surface, and slow down the rotation of the components, although Plavec (1970a) finds this argument uncertain] yet there are still difficulties. First, mass exchange in phase C can only occur in a wide system, and many Am stars are quite close systems. Second, many Am stars are found in two-spectra binaries containing stars with identical or closely similar spectra. Van den Heuvel suggests that in these systems the white dwarf is a third, invisible component. Such a component would not be detectable, in fact, even from its effect on the motion of the close pair, unless the highest spectrographic dispersion could be employed to observe the system. Although the hypothesis cannot be disproved, it should only be accepted if a strong case can be made for it on other grounds. My personal opinion is that this case has not yet been made, and that the existence of binaries containing two closely similar Am stars is a strong reason for rejecting the hypothesis.

Van den Heuvel's ideas about the Ap stars have been further elaborated by Guthrie, who argues that the peculiar abundances in the

atmospheres of these stars are consistent with the hypothesis of mass exchange, and indeed, cannot plausibly have been produced by any other means. This hypothesis would also seem to face a difficulty in the existence of binaries containing two similar Ap stars. Guthrie suggests that these were originally part of a triple system, the third component of which was lost in the process of mass exchange. Guthrie supposes that the mass exchange takes place during a supernova explosion, and that the peculiar abundances of the Ap stars are found only in a shallow surface layer.

Another speculative result of mass exchange has been discussed by Paczynski (1967c), Barbaro *et al.* (1969), and Kippenhahn (1969). This concerns the results of mass exchange in systems of total mass of about $30m_\odot$. The end product appears to be a luminous helium star, and Paczynski suggested the identification of these objects with the Wolf–Rayet components of binaries. The expected total mass of the system is of the right order, and the known periods of Wolf–Rayet binaries are also in accord with the calculations. The helium star itself is a fairly small object, as Wolf–Rayet stars are believed to be, from the work of Wilson (1942), Keeping (1947) and Kron and Gordon (1950) on V444 Cygni. Their results, discussed in more detail in Chapter 7, indicate that the star itself is quite small, although surrounded by a large envelope. It seems likely that the Wolf–Rayet stars are still losing mass, as is mentioned in Chapter 4. Paczynski has even attempted to explain the differences between the WC and WN spectra in terms of the amount of matter transferred in the mass exchange process. If so much is transferred that the outer layers of the convective core are exposed, these are likely to be deficient in nitrogen, but not in carbon. If less mass is transferred, the exposed layers will contain nitrogen. Against this hypothesis must be urged the possibility that Wolf–Rayet stars are in the stage of pre-main-sequence contraction (Sahade, 1958a) which is supported by the existence of similar binaries containing Of stars that might represent a later stage of the same process. There is also evidence (Underhill, 1966) that the large differences between WN and WC spectra are not the result of abundance differences, but of differing conditions of excitation in the Wolf–Rayet

atmosphere. Finally, Underhill has also urged (1969) that if the end product of mass exchange is to be convincingly identified with a Wolf–Rayet star, it is necessary to show that it will produce a Wolf–Rayet spectrum. Although this matter is still highly controversial, nevertheless Paczynski's suggestion is likely to prove fruitful because it will stimulate more observational and theoretical work in attempts to prove or disprove it.

ACHIEVEMENTS AND LIMITATIONS OF MASS-EXCHANGE THEORY

The principal achievement of the mass-exchange theory has been its successful qualitative explanation of the Algol-type systems as the results of mass exchange in phase A which are still in the stage of slow mass exchange. Not many systems are observed while they are in the rapid phase, because that phase *is* rapid. It lasts no more than 10^5 to 10^6 years for a primary of original mass $5m_\odot$, while the observable slow phase lasts 10^7 to 10^8 years in the same system. In addition, Plavec (1968) has shown that systems in which the primary component fills the Roche lobe are likely to be difficult to detect photometrically. Several authors have suggested that β Lyrae is the rare example of a system in the rapid phase of mass transfer. Perhaps the other massive systems classed by Sahade with β Lyrae are also in this phase. They are all characterized by underluminous secondary components. It is tempting to explain the underluminosity of these components as the direct consequence of mass exchange. Unfortunately, the evidence seems to be that it is the apparently normal primaries that are losing mass.

The fact that Algol-type systems have so far been only qualitatively explained by the theory may be held against it. As explained in the previous section, however, there are reasons for expecting that quantitative agreement cannot be very good at this stage. It is significant that when Paczynski and Ziółkowski relaxed the most artificial of the assumptions made in the theory (no loss of mass to the system as a whole) they were able to improve the agreement between theory and

observation. Except in a few cases, the observations do not help to decide whether mass is lost from the system or not. In Chapter 4 it is shown that period changes in either direction can be explained equally well by either loss or transfer of mass, and only if the escaping matter can be detected spectroscopically can its existence be ascertained. On the other hand, the energy considerations in the previous section give grounds for assuming that in the rapid phase *all* systems lose mass. There is not a unique velocity of escape from a system, because the direction, as well as the speed, of ejected matter determines whether or not it will escape. There is a minimum velocity that is sufficient but not necessary for escape. Plavec and Kříž (1965) estimate this to be just over 100 km/sec in RW Tauri, that is, of the same order as the velocities to be expected in the rapid phase of evolution. Although the critical velocity is higher for more massive systems, the loss of luminosity of the primary star is greater. The energy available for the ejected matter is therefore also greater.

The other applications of mass-exchange theory, discussed in the immediately preceding section, are clearly of a more speculative nature. It is tempting to explain the wide variety of existing binary systems as results of mass transfer from main-sequence objects in systems of different separations and mass ratios. These suggestions are fruitful in that they direct thinking and computation—it has been suggested that some shell stars are one result of mass exchange (Kříž, 1969; Plavec and Horn, 1969; Horn, 1970)—but some of the results must still be considered tentative. More will be known when the evolution of a complete grid of models has been computed, but this is time consuming, even with modern techniques, and has not yet been done. This, however, is a statement that may be out of date by the time this book appears in print.

Another major problem that has not yet been fully dealt with by computation is the evolution of the star receiving mass. Can it receive this amount of mass and angular momentum in periods of the order of 10^5 years or less, and still look like a main-sequence star? This problem would be eased if a considerable amount of mass loss from the system does occur in the rapid phase. Kruszewski (1967), however, argues that the star cannot absorb this mass and angular momentum immediately, and forms a ring in order to store the angular momentum until it can adjust to it. Gorbatskii finds that the rings in U Geminorum systems would collapse in about a month once the supply of new matter is cut off (1969). This suggests that their prime purpose is not storage of angular momentum, since a month is a negligible time, even when compared with phases of rapid change in a star. Systems like U Geminorum may differ in this respect from others, but the mass stored in the ring of a typical Algol primary, at any given time, must be a very small fraction of the total being transferred. The spectrum of U Cephei, as described in Chapter 8, is not precisely that of a main-sequence star of spectral type late B. Giannone and Weigert (1967) did discuss the evolution of a mass-receiving star that is already a white dwarf, and Lauterborn (1968) suggested that U Geminorum outbursts might be related to an increasing infall of matter on the blue star. Some theoretical work by Paczynski *et al.* (1969) and Bath (1969) lends further support to this view, but detailed computations of the effects of "mass reception" are yet to be undertaken. [An abstract of such a paper has recently been published (Benson, 1970).]

Another problem for the theory is the place of W Ursae Majoris systems within it. There has been no real explanation for them yet, although Ziółkowski (1969) has suggested that they will form under certain conditions from mass transfer in the phase AB. Again, the system TX Cancri, apparently unevolved, is a stumbling-block to this theory of the origin of these systems. Kraft (1967) has suggested an evolutionary relation between the W Ursae Majoris systems and the U Geminorum systems. This, however, remains conjectural. The group of systems to which SX Cassiopeiae belongs also seems to represent a problem for the theory. These systems show considerable evidence of circumstellar matter, and yet neither star appears to have filled its Roche lobe.

It is of interest to consider possible alternative explanations for the Algol-type systems. Koch (1970) discussed RZ Cancri, and suggested that it *could* be explained on the assumption that it is a young system in which the secondary star is still contracting to the main sequence.

This is another suggestion that has value because of the work it is likely to stimulate in attempts to prove or disprove it. It encounters difficulties that were pointed out by Koch himself, and it could hardly be used to explain all the Algol-type systems. The low values of k_{22} found for three secondary components of Algol-type systems (Chapter 6) indicate that these are not contracting stars still to reach the main sequence. Zahn (1966) has also suggested that tidal effects could cause the more massive star to be more thoroughly mixed—thus giving it a longer main-sequence lifetime than its less massive companion would have, as that would expand when only its core hydrogen was burnt. Zahn's hypothesis suffered, perhaps, from being presented at the same conference as the original communication by Kippenhahn and Weigert on mass transfer. It should not be completely overlooked, however, because it served as a reminder that tidal effects may have an important bearing on evolution, and their consideration may be useful in improving the agreement between theory and observation.

Although the theory of mass exchange in close binaries is not yet completely satisfactory, its achievements are certainly spectacular. Unless the modern theory of stellar evolution is completely wrong, the components of binary systems must expand until they become unstable, and they must then lose mass, either to their companions, or from the system altogether. The most promising approach to a further understanding of binary stars, therefore, appears to be to develop this theory still further, relaxing as many of the present assumptions as possible, and introducing subsidiary effects (tidal forces, rotation) into the model computations.

EVOLUTION OF STARS IN VISUAL BINARIES

Although many visual binaries may have to be considered as "close" binaries in the sense defined in Chapter 1, undoubtedly some will be wide pairs. If the system of Sirius, with its orbital period of 50 years, can be seriously considered as the result of mass exchange, then many of the visual binary systems that appear in the orbit catalogues may also be close binary systems. In particular, it may be surmised that all visual binaries that contain white dwarfs are close pairs that once contained a massive star that has evolved right through the mass-exchange process. It is not possible to assign a minimum orbital period such that all systems having longer periods are necessarily wide systems, unless more is known about the masses and luminosities of stars in wide visual pairs. Most common-proper-motion pairs are presumably wide, and many of the pairs recorded in the *Index Catalogue* are probably genuinely wide pairs. Even amongst the pairs that show orbital motion, there will be many that have not yet gone through mass exchange, because observational selection results in the known pairs of this type being primarily those containing stars at the lower end of the main sequence, which will not exhaust their hydrogen for a long time to come. If pairs containing white dwarfs are excluded, therefore, a sample of visual binaries will be overwhelmingly composed of stars that have not yet been, and perhaps will not ever be, involved in mass exchange, except possibly during their pre-main-sequence lifetime.

Wide binaries of this kind have frequently been used as tests for the theory of stellar evolution. It is even more likely that their components are of the same age than that the members of a cluster are. If mass has not been exchanged between the components, therefore, no pairs of stars should be found in which the less massive component is the more evolved. Investigations of visual pairs have been made for this purpose by Bidelman (1958), Stephenson (1960), Petrie and Batten (1964), Batten (1966), and Giannone and Giannuzzi (1969). The chief difficulty is that pairs well enough resolved for good data (spectrograms, photoelectric colours, and magnitudes) to be obtained are often the pairs for which relative orbital motion is slow or non-existent, and for which, therefore, there is much doubt as to the reality of a physical connection between the two. Even a common-proper-motion pair, however, is very likely to have had a common origin. The conclusion of most investigators so far is that the overwhelming majority of visual binaries do support the current theory of stellar evolution. One exception, pointed out by Stephenson, is 95 Herculis, in which the less luminous star of the pair appears well to the right of the main sequence in the

Hertzsprung–Russell diagram although its companion is on the main sequence. The secondary star may still be contracting towards the main sequence: lines of lithium have been detected in its spectrum (Wallerstein, 1965). The light of one of the stars may be variable (Muller, 1952). There are a few other systems that are possibly anomalous, but it is not so certain that these are genuine binary systems, rather than optical doubles.

THE ORIGIN OF BINARY STARS

To discuss the evolution of binary stars at some considerable length, before even mentioning their origin, may appear to be putting the cart before the horse. Although much progress has been made in the last few years towards understanding evolution, little has been made on the problem of the origins of binary systems. Even the origin of the orbital angular momentum is obscure. Most investigations that have been made of the orientation of the orbital planes of visual binaries indicate that this is random. The orbital angular momentum cannot, therefore, be derived from galactic rotation. Indeed, Dommanget (1968) finds that orbital planes parallel to the galactic plane seem to be avoided. Huang and Wade (1966) have found, statistically, that the orbital planes of eclipsing binaries are also oriented at random. Ostriker (1970) has pointed out that a condensing protostar will derive considerable angular momentum from the random motions of the condensations in the prestellar cloud, and this poses a problem for the formation of single stars, although Prentice and ter Haar (1971) find that this may be avoided if stars condense from dust clouds rather than gas couds. It is thus possible that the formation of multiple systems is an essential step in the formation of single stars. The high observed frequency of multiple systems and the random orientation of angular-momentum vectors are then to be expected.

A general comment can be made about all theories of the origin of binary systems. Any theory must be able to explain the observed proportion of multiple systems. It is possible, of course, that binary systems are produced in more than one way. One way might produce

only binary systems, since they occur in larger numbers than are expected if one process is responsible for both binary and multiple systems (Chapter 3). Nevertheless, a theory that does not account for the production of multiple systems in appreciable numbers cannot be considered as a theory of the origin of *all* binaries.

Four plausible hypotheses of the origin of binary stars have been put forward.

First, there is the capture hypothesis. Two stars are supposed to have become gravitationally bound as the result of a chance encounter. A triple encounter is needed, or at least the influence of the local interstellar medium, otherwise the encounter will not result in a capture. This requirement makes the probability of capture very low, at least in regions with an average stellar density comparable with that in the neighbourhood of the Sun. Ambartsumian (1937) has argued that the observed frequency of wide visual pairs is far too large to be accounted for by the capture hypothesis, since there must be equilibrium between the newly formed pairs and those which dissociate as a result of encounters. Although some triple systems may form from a close encounter between an already existing double star and a third body, it seems very unlikely that the observed function $f(n)$ (Chapter 3) could be reproduced by this mechanism. The capture hypothesis would probably also require a very different distribution of semi-axes from that actually observed, because systems of large negative energy (i.e. close pairs) are unlikely to result from chance encounters. This question could probably be profitably investigated by modern computing techniques. If binaries are formed by capture, the two stars in a system are not necessarily of the same age. The observed result that only a very few pairs, at most, are found to contradict the assumption that the components are of the same age is, therefore, a legitimate argument against the capture hypothesis as the origin of a substantial number of systems. Some systems may have been formed in this way in the early history of clusters and associations.

Second, there is the hypothesis that double stars evolved by fission of a single rapidly rotating star. The classical treatment of fission theory has been criticized by several investigators because it applies

only to incompressible fluids (although, according to Ostriker, 1970, this criticism rests on a partial misunderstanding of the assumptions made in the early investigations). Ambartsumian (1956) has pointed out that the orbital angular momentum of any binary system exceeds by several orders of magnitude the rotational angular momentum of its components. Thus, any known single star that is rotating rapidly has far too little angular momentum within it to form a binary system. The fission theory also has great difficulty accounting for the existence of wide pairs. Ambartsumian (1937), Chandrasekhar (1944) and Yabushita (1966) have all investigated the possibility that encounters of binary systems with other stars will gradually separate, and eventually dissociate, binary pairs. They all agree that even a binary with an initial separation of 100 A.U. will have a mean lifetime of the order of 10^{12} years, so no great changes in separation are to be expected in the whole main-sequence lifetime of even low-mass components (10^9 to 10^{10} years). A system newly formed by fission would be much more tightly bound. The fission hypothesis makes the existence of multiple systems, at least in the observed proportion, appear very unlikely.

Despite all these difficulties, there is one group of systems so strongly suggestive of fission that the hypothesis is constantly revived to explain it. This is the group of W Ursae Majoris systems. The angular momentum problem is less difficult to overcome for this group, because the orbital momentum exceeds the rotational by a smaller factor than in any other group of binary systems. Until recently this factor was supposed to be 4 or 5 (Struve, 1950), but Smak (1964b) has advanced reasons for supposing that it may be a hundred or more. The assumptions that he has made in order to calculate the rotational momentum (chiefly, rigid body rotation) tend to lead to an overestimate. If Smak's results are correct, fission is an improbable origin, even for the W Ursae Majoris systems. The most interesting recent revival of the fission theory was by Roxburgh (1965, 1966 a, b). He supposed that during the stage of a protostar's life in which a radiative core begins to form and grow, the central regions become uncoupled from the outer envelope and continue to contract, conserving their angular momentum, while the surface is losing mass through rotational break-up. The contracting core is virtually incompressible, so fission can occur there. He found that fission could occur if the original total mass of the protostar was between $0.8m_\odot$ and $4.0m_\odot$, and this compares very favourably with the limits of $0.74m_\odot$ and $3.8m_\odot$ between which the total masses of W Ursae Majoris systems listed by Kopal and Shapley (1956) fall. These masses are very difficult to determine, however, and consequently uncertain. Moreover, as is discussed in Chapter 7, it is not entirely clear whether "contact" systems of higher total mass belong to a separate group, or are essentially similar to the W Ursae Majoris systems. I have critically discussed the observational tests of Roxburgh's theory more fully elsewhere (1967). Almost at the same time, Huang (1967) advanced a more fundamental objection to it. The core and envelope cannot remain uncoupled once the core has become appreciably distorted, and as this uncoupling is fundamental to the fission, it is unlikely that fission will take place.

Other investigators have recently found evidence in favour of the fission theory. Ostriker (1970) believes that some of the criticisms levelled at earlier versions of the fission theory are not altogether valid. Steinetz and Pyper (1970) have found a correlation between the rotational velocities of the components of wide visual binaries, which, they suggest, could be a result of fission. This, however, is just the group of systems for which fission seems least likely on other grounds. The possibility that some kinds of binaries have been formed through fission cannot be entirely ruled out, but it seems unlikely that this hypothesis can explain the formation of all binaries.

The third hypothesis to consider is that binary stars are formed by condensation of the interstellar cloud about separate but neighbouring nuclei. Wood (1964) made an interesting qualitative investigation of this possibility, that may well be a useful starting-point for future work. The main problem that he was trying to explain, however (the existence of Algol-type systems), has now been more convincingly explained by mass-exchange theory. Huang (1957) has also tried to develop a theory along these lines. If the process of condensation of stars takes place randomly, then the distribution of multiple systems

according to the number, n, of their components [the function $f(n)$] should be a Poisson distribution (Chapter 3). It is shown that the observed $f(n)$ approximates to such a distribution except for an excess of binary systems. Of the hypotheses discussed so far, this is the only one that can account for the existence of multiple systems in something like the observed proportions. In Chapter 3, however, the Poisson distribution is arrived at by dividing the Galaxy into cells of such a size that they contain, on average, one star. This is a fairly large cell when compared with the average separation of a spectroscopic binary. A major difficulty for this theory is that the formation of one condensation in a prestellar medium tends to inhibit the formation of any nearby ones. Moreover, the initial protostars are very large. It seems very unlikely that a close binary can be formed in this way with the components already close. Could a system like A.D.S. 9537—a widely separated common-proper-motion pair, each member of which is a W Ursae Majoris system (Batten and Hardie, 1965)—have been formed in this way?

The fission theory required a process for increasing the separation of binary stars, and no effective mechanism has yet been found. The independent-condensation theory appears to need a process for decreasing the separations of binary components. Huang (1966c) and Mestel (1967) independently suggested the process of magnetic braking, first proposed by Schatzman (1962), might be able to do this. Ionized particles ejected by either star and constrained to travel along lines of force will acquire considerable angular momentum that they will take away from the system, should they actually escape from it. The separation of the component stars will therefore eventually decrease. This however, postulates a coupling between the orbital and rotational angular momenta. If the two stars are magnetically linked, then angular momentum can be taken out of the system directly, even if the two stars are not dynamically coupled. Huang finds that the mechanism is likely to be effective for separations up to 100 times the radii of the stars, or possibly even 1000 times. Since he supposes that it operates in the pre-main-sequence phase of contraction, when radii are large, he is able to suggest that it is a mechanism capable of bringing together the components of quite widely separated binaries. In Huang's view, the W Ursae Majoris systems are not the first stages of a fission process, but the last stages of a fusion process. According to Mestel the whole process of bringing the binary components together may be completed in times of the order of 10^3 years. He even suggests that a single rapidly rotating B star may be formed in this way.

The fourth theory that should be considered was recently discussed by van Albada (1968b) and also Worrall (1967). This theory is in some respects an extension of the theory of separate condensations. It is supposed that stars form within clusters in small groups containing up to some tens of members. These groups are unstable and disintegrate in from 10^3 to 10^8 years into multiple systems centred on a "close" binary ("close" in this context means a separation of about 10 A.U.). The number of stars left in the final multiple system depends on the number in the original group. A similar hypothesis has been discussed by Szebehly (1969) who has investigated the effect of close encounters between members of a triple system. The encounters enable one star to escape, leaving behind a much closer double than was found in the original system. This hypothesis, like the previous one, links the formation of binary and multiple systems. The ratio of multiple to binary systems predicted by it depends on the frequency distribution of the number of members in the original groups from which the multiple systems are supposed to descend. On general grounds, it seems likely that this distribution should be a Poisson distribution, and therefore the frequency distribution of systems of different multiplicity is likely to be related to the Poisson distribution. On the other hand, as van Albada points out, the hypothesis offers no explanation for very close binaries, at least in the form in which he presents it. Perhaps it can be combined with the theory of magnetic braking to offer an explanation of all kinds of binary and multiple systems.

Blaauw and van Albada (1967) have recently suggested that binary systems are produced in more than one way. They have investigated the frequency of binaries, as a function of the major semi-axis of their orbits, in a number of clusters and associations of early-type stars.

They find that the frequency distribution has two maxima—one corresponding to short-period spectroscopic binaries, the other to visual binaries with separations of the order of 100 A.U. There are many more visual binaries than would be expected from the frequency distribution of the spectroscopic binaries. Blaauw and van Albada suggest that each group of binaries has been formed in a different way. This idea is very attractive, because it eliminates the need to find a way to turn close binaries into wide ones, or vice versa. It was first suggested by Bleksley (1934). One argument used against it was the existence of a period-eccentricity relation, which used to be believed to be single and continuous for all kinds of binary system. There seems to be a large measure of agreement now that the apparent relation found for visual binaries is a result of observational selection (see Heintz, 1969, for a discussion). The situation is not so clear for spectroscopic binaries. Kruszewski (1966) has tried to reproduce such a relation from mass-exchange computations, and Zahn (1966) has shown that tidal interactions in close binaries tend to reduce orbital eccentricities. Bouigue *et al.* (1967) have found little more evidence of a period-eccentricity relation than that there is an upper limit to the eccentricity for any given period. Such a limit must obviously exist, otherwise tidal forces acting at periastron would speedily alter the system. A more complicated relation was found by Walter, however (1950). He found two distinct period-eccentricity relations for spectroscopic binaries, provided the systems were classified according to the spectral type of the primary component. He also explained the relations by a theory involving tidal interactions between the components. The existence of a unique period-eccentricity relation for all classes of binaries seems, however, highly unlikely, and it no longer constitutes an argument against the hypothesis of different modes of origin for binaries of different types. A possible argument against the form of the hypothesis put forward by Blaauw and van Albada is that selection effects produce an artificial minimum in the frequency distribution of the semi-axes. They have taken great care to avoid selection in the discovery of spectroscopic binaries, however, and claim completeness down to $K_1 = 15$ km/sec. Since they find *more* visual binaries than they

expected from the frequency distribution of spectroscopic binaries, it seems unlikely that their discoveries of visual binaries are seriously incomplete. Nevertheless, the interval of semi-axes for which they find a minimum is just the interval in which either spectroscopic or visual binaries would be hard to detect. Their results refer only to three or four clusters and may not apply generally. The possibility that at least two modes of formation of binary systems are operating cannot be ruled out. The possible excess of binary systems over the prediction of a Poisson distribution is additional support for it. It might be noted that although mass exchange within a binary system can provide a means of changing the separation, it cannot transform a visual binary into a spectroscopic binary, or vice versa, because after the initially more massive component has become the less massive, the semi-axis begins to return to its initial value.

The problem of origins is bound to remain mysterious. We cannot watch a binary star form, and we might not recognize it if we could. Most discussions of the problem, at present, are qualitative, and sometimes vague. This discussion is no exception. It is premature to draw conclusions, but it is possible to delineate the problem in such a way as to show what must and can be done to reduce the area of mystery. Each of the processes discussed can provide definite predictions about the distribution of multiple systems $f(n)$, and of semi-axes $\Phi(a)$. At least one of these may change with time, and there may be differences between field stars and members of clusters. If these functions were computed theoretically under definite assumptions, the theories would be easier to test. On the other hand, the observational determination of these functions must be vastly improved. The functions are difficult to determine because there are so many selection effects, but the crude estimates of $f(n)$ given in this book can almost certainly be improved, as can the pioneer work on $\Phi(a)$ (Kuiper, 1935). In these ways we can hope to obtain a better understanding of the origin of binary and multiple systems. Since it appears probable that more stars are to be found in such systems than outside them, any progress in understanding such systems is important progress for the whole of astronomy.

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